FOURIER ANALYSIS

Reminders and prerequisite exercises

Differential equations.

1. Let $\nu > 0$. The solutions of the ordinary differential equation $y'' - \nu^2 y = 0$ on the line form a vector space of dimension 2. Find

(1) a basis of the solution space;
(2) a basis f, g with $f(0) = 0$;
(3) a basis f, g with $f(0) = 0$ and $g(\ell) = 0$, where $\ell > 0$ is given.

2. Do the preceding exercise, but now with the equation $y'' + \nu^2 y = 0$. When is there a nontrivial solution y with $y(0) = y(\ell) = 0$, and then what about part c)?

Trigonometric functions.

1. If k is integer, $\cos(k\pi)$ is either $+1$ or -1. Verify that $\cos(k\pi) = (-1)^k$. Find a similar expression for $\sin((k - \frac{1}{2})\pi)$.

Recall some trigonometric formulas like

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \text{ and } \cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y),$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2} \text{ and } \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

and the "product formulas"

$$2\sin(x)\cos(y) = \sin(x+y) + \sin(x-y)$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

$$2\sin(x)\sin(y) = -cos(x+y) + \cos(x-y).$$

2. Find a primitive of the function $f(x) = \sin(ax)\sin(bx)$ for nonzero a and b.

Some other functions.

Recall the definitions of the functions sinh, cosh and tanh and sketch their graphs. Some more functions:

$$\sec(x) = \frac{1}{\sin(x)}, \text{ and } \csc(x) = \frac{1}{\cos(x)}.$$