Tentamen i integrationsteori 2007–08–29 08.30-13.30
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1. Formulate and prove Fatou's lemma.

2. Formulate and prove the monotone convergenece theorem.

3. For f in L^1, let $\|f\| = \int |f|$.

   a Show that if
   $$\sum \|f_j\| < \infty,$$
   then
   $$\sum f_j(x)$$
   is convergent for almost all x.

   b Show that if $\{f_j\}$ is a Cauchy sequence in L^1 (i e $\|f_j - f_k\| \leftarrow 0$), then there is a subsequence $\{f_{j_m}\}$ which converges almost everywhere and in L^1.

   c Show that any Cauchy sequence in L^1 is convergent.

4. Prove that if f and g are real valued functions, then
   $$(\int f)^2 + (\int g)^2 \leq (\int \sqrt{f^2 + g^2})^2.$$
   Then prove the corresponding statement for 3, 4,...N functions.

5. Is there a measurable subset E of $[0, 1]$ such that
   $$m(E \cap (a, b)) = (b - a)/2$$
   for $0 < a < b < 1$?

6. Show that
   $$\int (\int_{0<y<f(x)} f(x)dx)dy = \int_{f>0} f^2(x)dx.$$

7. Let $g(x)$ be a continuous function on R which is periodic with period 1. Assume
   $$\int_0^1 g = 0.$$

   a Show that if f is continuously differentiable on the interval $[0, 1]$ then
   $$\lim_{n\to\infty} \int_0^1 f(x)g(nx)dx = 0.$$

   b Show that the same thing holds for any f in L^1 on $[0, 1]$.