Tentamen i integrationsteori 2008–01–14 08.30-13.30
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BB

1. Formulate and prove the monotone convergence theorem.

2. Prove (and state precisely) Fubini's theorem for measurable sets in a product space
$$\mu \times \nu(E) = \int \nu(E_x) d\mu(x) = \int \mu(E^y) d\nu(y).$$

3. State and prove the Hardy-Littlewood maximal theorem.

4. Assume $\mu(X) < \infty$ and let $f \geq 0$ be a measurable function such that
$$\int f^2 d\mu < \infty.$$
Show
$$\int f d\mu < \infty.$$
2

5. Suppose f is integrable on $(0, \infty)$. Show that
$$\sum |f(2^n x)| < \infty$$
almost everywhere. 2

6. For a measurable real valued function f on a measure space X, put
$$E(t) = \mu(\{x; |f(x)| > t\}).$$
a) Prove that
$$\int |f|^2 d\mu = \int_0^\infty p t^{p-1} E(t) dt.$$
2
b) Suppose $|f| \leq 1$ and that $E(t) \leq C/t$. Prove that
$$\int |f|^2 d\mu < \infty.$$
2

7. Let
$$I_n(g) = \frac{2n}{\sqrt{\pi}} \int_0^\infty e^{-(nx)^2} g(x) dx.$$
a) Assume g is bounded, measurable and continuous at the origin. Show that $I_n(g) \to g(0)$ as n tends to infinity. 2 b) Assume g is in L^1 and put
$$G(x) = \int_0^x g(t) dt.$$
Assume $G'(0) = g(0)$. Show that $I_n(g) \to g(0)$ as n tends to infinity. (Hint: integrate by parts) 2