MATEMATIK
Chalmers tekniska högskola
Tentamen

Hjälpmedel: Inga, inte ens räknedosa
Telefon: Namn, ankn. ????
199?–??–?? kl. ??.??–??.??

Kurs

OBS! Ange kod, kurskod samt linje.

MMA210, Advanced Differential Analysis
Assignment 1 , turn in: Course-week 3

1. Suppose that the function (mapping) f possesses the following property: It maps any open set into an open set ($f(U)$ is open for any open U.) Need f to be continuous? Suppose additionally that f is injective (the image of any point is one point.) Is f necessarily continuous?

2. Let f be a continuous mapping, $f : S^1 \to S^1$, where S^1 is the unit circle in the complex plane, $S^1 = \{z \in \mathbb{C}, |z| = 1\} = \{e^{i\theta}, 0 \le \theta \le 2\pi\}$. Prove that there exists a continuous function $g : \mathbb{R} \to \mathbb{R}$ such that $g(\theta + 2\pi) = g(\theta) + 2\pi k$ for some integer k satisfying

$$f(e^{\imath theta}) = e^{\imath g(\theta)}.$$

Hint. Study some particular case, say, $f(z) = z^2$. Try constructing g first locally, and then use the compactness of S^1.