Integrality

The inclusion $\mathbb{Z}[i] \subset \mathbb{Q}(i)$ and its characterization in Prop. 1.5 are prototypical for the concept of integrality.

An alg. integer is a zero of a monic polynomial over $\mathbb{Z}$.

Aud. Find a monic polynomials over $\mathbb{Z}$ with $i+1$, $\dfrac{1+\sqrt{-3}}{2}$, and $\dfrac{1+\sqrt{-7}}{2}$ as their zeros.

Is $\dfrac{1+\sqrt{-5}}{2}$ an alg. integer?

We throughout work with unital, commutative rings.

Prop 2.2, 2.3.

Aud: Sketch proof of 2.3.

Proof of 2.2.

Aud: Give $\mathbb{Z}$-module generators of $\mathbb{Z}\left[\frac{1+\sqrt{-3}}{2}\right]$ and $\mathbb{Z}\left[\frac{1+\sqrt{-5}}{2}\right]$.

Sums and products of integral elts are integral.
Prop. 2.4. and proof.
Integral closure and normalization.
Factorial => normal.
Aud: Show that $\mathbb{Z}[\sqrt{-3}]$ is not euclidean.

Extension of fields and integers; minpolys.
Def 2.5: Trace, norm.
give homomorphisms
Prop 2.6: Galois theoretic interpretation; proof.
Aud: Fix a $\mathbb{Z}$-basis of $\mathbb{Z}[\sqrt{-1}]$ and $\mathbb{Z}\left[\frac{1+\sqrt{-3}}{2}\right]$ and give $T_{\sqrt{-1}}$ and $T_{\sqrt{-3}}$ in coordinates.

Cor 2.7: Separable case and proof.

Aud: What is the smallest inseparable extension?

Aud: Is the extension $\mathbb{F}_2((t)) \subseteq \mathbb{F}_2(t)$ separable?

Cor 2.7: Inseparable case without proof.

Rm: Char 0 => perfect.

Discriminant of a ~~field~~ field extension.

Aud: ~~Proof~~ Existence of primitive elements for number fields.

Vandermonde interpretation

Prop 2.8 and proof.

Tr and N of integral elements, units.

Lemma 2.9 and proof

~~integral basis~~

Aud: Compute disc($\mathbb{Z}[\sqrt{-3}]$) and verify Lm 2.9 directly.

integral basis and existence

Prop 2.10 and proof.

Aud. Find ~~the~~ a basis for the integers in $\mathbb{Q}(\sqrt{-1}, \sqrt{-3})$.

Prop 2.11 and proof, and remark.

~~Aud: Find an integral basis for $\mathbb{Q}(\sqrt{-3}, \sqrt{-7})$.~~

Discriminant of $\mathcal{O}_K$ modules.

Prop 2.12 and proof

For ex. session 1: Ex2
2: Ex.4
3: Ex.5
1: Ex.7.