Ideals

Introduction from book. Aud Ex 1.
~~Define: ideals.~~
Aud: Show that $\mathfrak{a}$ is an ideal when following the division rules.

$\mathfrak{a} \mid \mathfrak{b} \iff \mathfrak{b} \subseteq \mathfrak{a}$.

Noetherian: every ideal fl. gen.;
equivalently: Every descending chain of ideals stabilizes.

Aud Show equivalence.

Prime ideal: $ab \in \mathfrak{p} \Rightarrow a \in \mathfrak{p} \vee b \in \mathfrak{p}$.

Maximal ideal: $\mathfrak{m} \subsetneq O_K$ maximal.

Aud: Rings $A \subseteq B$. $\mathfrak{p} \subseteq B$ prime ideal.
$\Rightarrow A \cap \mathfrak{p}$ prime ideal.

Aud: k field, α algebraic., ~~then~~
~~$k[\alpha]$ a field direct sum of fields.~~
If $k[\alpha]$ an integral domain, then $k[\alpha]$ a field.

Thm 3.1 and proof. ; And Ex. 3

Def. 3.2.

Define PID, revise gcd, lcm.

Explanation as in book for extensions of PIDs.

State Thm 3.3.

Lemma 3.4.

Lemma 3.5

And: In a PID, ~~see~~ inspect how the proof would go. (i.e. $\mathfrak{p} = (a)$).

Pf of 3.3: Existence

And: Factor the ideal (6) in $\mathbb{Z}[\sqrt{-5}]$.

Pf of 3.3: Uniqueness.

Product decompositions as in book.

And Spell out $\mathfrak{a} = \bigcap \mathfrak{a}_i = \prod \mathfrak{a}_i$ for coprime $\mathfrak{a}_i$.

(1) $\mathfrak{a}, \mathfrak{b}, \mathfrak{c}$ coprime $\Rightarrow \mathfrak{ab}, \mathfrak{c}$ coprime; $\mathfrak{a}, \mathfrak{b}$ coprime, $h \in \mathfrak{a}, \mathfrak{b}$ $\Rightarrow h \in \mathfrak{ab}$.

Thm 3.6 and proof

<u>Aud</u> Ex 4, 5, 6.

Def 3.7

Generators $\frac{a_i}{b_i}$ of $\mathfrak{a}$. $\Rightarrow$ $\underbrace{\prod b_i}_{\neq 0} \mathfrak{a} \subseteq \mathcal{O}$.

$\mathfrak{a} \subseteq \mathcal{O}$ ideal $\Rightarrow$ f.d. gen.

$\Rightarrow \frac{1}{c}\mathfrak{a}$ is a fractional ideal.

<u>Aud</u> Multiply $(2, 1+\sqrt{-5})$ and $(3, 1+\sqrt{-5})$ in $\mathbb{Q}(\sqrt{5})$; give two generators.

Prop 3.8 and proof.

Cor. 3.9.

<u>Aud</u> Factor the fr. ideal $(\frac{1}{6})$ in $\mathbb{Q}(\sqrt{-5})$

Exact sequence of class group.