1. $0 = x^3-6x^2+11x-6 = (x-1)(ax^2+bx+c) = (x-1)(x^2+bx+6) = (x-1)(x^2-5x+6) = (x-1)(x-2)(x-3)$ så rötterna är alltså $x_{1,2,3} = 1,\ 2,\ 3$.

---

2. $p(x) = -3x^2+9x-12 = -3(x^2-3x+4) = -3(x^2+2(\frac{-3}{2})x+4) = -3((x+(-\frac{3}{2}))^2-(-\frac{3}{2})^2+4) = -3((x+(-\frac{3}{2}))^2-\frac{9}{4}+\frac{16}{4}) = -3((x+(-\frac{3}{2}))^2+\frac{7}{4}) = -3(x+(-\frac{3}{2}))^2-\frac{21}{4} \leq -\frac{21}{4}$ så $x = 3/2$ är en maximipunkt med maximum $-21/4$.

---

3. $x+3 \geq \frac{2x}{x-2} \Leftrightarrow x+3-\frac{2x}{x-2} \geq 0 \Leftrightarrow \frac{(x+3)(x-2)-2x}{x-2} \geq 0 \Leftrightarrow \frac{x^2-x-6}{x-2} \geq 0 \Leftrightarrow \frac{(x+2)(x-3)}{x-2} \geq 0$ Teckenstudietabell ger:

x		-2		2		3	
$x+2$	$-$	0	$+$		$+$		$+$
$x-2$	$-$		$-$	0	$+$		$+$
$x-3$	$-$		$-$		$-$	0	$+$
$\frac{(x+2)(x-3)}{x-2}$	$-$	0	$+$	ej def	$-$	0	$+$

vilken ger svaret $-2 \leq x < 2$ eller $x \geq 3$.

---

4. $\cos 2v = \sin v = \cos(\frac{\pi}{2}-v) \equiv \alpha$ Om ω_0 är en lösning till ekvationen $\cos\omega = \alpha$ så ges alla andra lösningar ω av att $\omega = \pm\omega_0 + n2\pi$. Detta ger två fall: i) $2v = \frac{\pi}{2}-v+n2\pi$ och ii) $2v = -(\frac{\pi}{2}-v)+n2\pi$. Fallet i) har lösningar $v = \frac{\pi}{6}+n\frac{2\pi}{3} \Leftrightarrow v = \frac{\pi}{6}+n2\pi$ eller $v = \frac{5\pi}{6}+n2\pi$ eller $v = \frac{3\pi}{2}+n2\pi$. Fallet ii) har lösningar $v = -\frac{\pi}{2}+n2\pi$ och dessa ingår ju i beskrivningen $v = \frac{3\pi}{2}+n2\pi$ i fall i). Alltså är lösningarna $v = \frac{\pi}{6}+n\frac{2\pi}{3}$.

---

5. **a)** $D(\sin x^2) = \cos(x^2)D(x^2) = 2x\cos(x^2)$ **b)** $D(\frac{x^2}{e^{3x^2}}) = \frac{2xe^{3x^2}-x^2e^{3x^2}6x}{(e^{3x^2})^2} = \frac{2x(1-3x^2)}{e^{3x^2}}$ **c)** $D(\ln(x+\cos^2 x) = \frac{1}{x+\cos^2 x}D(x+\cos^2 x) = \frac{1-2\cos x\sin x}{x+\cos^2 x} = \frac{1-\sin 2x}{x+\cos^2 x}$ **d)** $D(\tan\sin x^2) = (1/\cos^2(\sin x^2))D(\sin x^2) =$
$= (1/\cos^2(\sin x^2))(\cos x^2)D(x^2) = (1/\cos^2(\sin x^2))(\cos x^2)2x = \frac{2x\cos x^2}{\cos^2(\sin x^2)}$ **e)** $D((1+\cos x)^{1/x}) =$
$= D(e^{(\ln(1+\cos x)^{1/x})}) = D(e^{(\ln(1+\cos x))/x}) = e^{(\ln(1+\cos x))/x}D((\ln(1+\cos x))/x) = (1+\cos x)^{1/x}D((\ln(1+\cos x))/x) = (1+\cos x)^{1/x}[(1/(1+\cos x))D(1+\cos x)x-\ln((1+\cos x))]/x^2 = (1+\cos x)^{1/x}[(1/(1+\cos x))(-\sin x)x-\ln(1+\cos x)]/x^2 = -(1+\cos x)^{1/x}[x\sin x+(1+\cos x)\ln(1+\cos x)]/(x^2(1+\cos x))$

---

6. **a)** $\int xe^{x^2}\,dx = \int\frac{d}{dx}(\frac{1}{2}e^{x^2})\,dx = \frac{1}{2}e^{x^2}+C$ **b)** $\int x\ln x\,dx = [PI] = \frac{x^2}{2}\ln x - \int\frac{x^2}{2}\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}+C$ **c)** $\int\cos\sqrt{x}\,dx = [t=\sqrt{x},\ x=t^2,\ dx=2tdt] = 2\int t\cos t\,dt = [PI] = 2(t\sin t - \int\sin t\,dt) = 2(t\sin t+\cos t+C) = 2(\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}+C) = 2\sqrt{x}\sin\sqrt{x}+2\cos\sqrt{x}+C$ **d)** $\frac{1}{x^2(x-4)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-4} = \frac{Ax(x-4)+B(x-4)+Cx^2}{x^2(x-4)} \Rightarrow A = -\frac{1}{16},\ B = -\frac{1}{4},\ C = \frac{1}{16}$. Vi får $\int_1^2\frac{dx}{x^2(x-4)}\,dx = \int_1^2 -\frac{1}{16}\frac{1}{x}-\frac{1}{4}\frac{1}{x^2}+\frac{1}{16}\frac{1}{x-4}\,dx = [-\frac{1}{16}\ln|x|+\frac{1}{4}\frac{1}{x}+\frac{1}{16}\ln|x-4|]_1^2 = -\frac{1}{16}(\ln 2-\ln 1)+\frac{1}{4}(\frac{1}{2}-1)+\frac{1}{16(\ln 2-\ln 3)} = -\frac{1}{16}(2+\ln 3)$.

---

7. **a)** $\lim_{x\to\infty}\frac{e^{2x}(1-5e^{-x})}{(e^{4x})^{1/2}\sqrt{1-3e^{-4x}}} = \lim_{x\to\infty}\frac{1-5e^{-x}}{\sqrt{1-3e^{-4x}}} = \frac{1}{1} = 1$ **b)** $\lim_{x\to 0} e^{\frac{1}{3x^2}\ln(x^2+\cos^2 x} = (\ell\text{'Hospital}) =$
$= e^{\lim_{x\to 0}\frac{\frac{1}{x^2+\cos^2 x}(2x+2\cos x(-\sin x))}{6x}} = e^{\lim_{x\to 0}\frac{1}{x^2+\cos^2 x}\lim_{x\to 0}\frac{2x+2\cos x(-\sin x)}{6x}} = e^{\lim_{x\to 0}\frac{2x+2\cos x(-\sin x)}{6x}} = (\ell\text{'Hospital}) =$
$= e^{\lim_{x\to 0}\frac{2-2(\cos^2 x-\sin^2 x)}{6}} = e^{\frac{2-2(1-0)}{6}} = e^0 = 1$ **c)** Ekvationen är 1:a ordningens linjär med IF $e^{-x^2/2}$ och lösning $y = -1+Ce^{x^2/2}$ där begynnelsevillkoret ger $y = -1+2e^{x^2/2}$. Då gäller alltså $\lim_{x\to 0}\frac{y-1}{x^2} = \lim_{x\to 0}\frac{2(-1+e^{x^2/2})}{x^2} = (\ell\text{'Hospital}) = \lim_{x\to 0}\frac{2(xe^{x^2/2})}{2x} = \lim_{x\to 0}e^{x^2/2} = e^0 = 1$

---

8. Se kursboken för ett bevis av integralkalkylens huvudsats.
