1a) $\frac{32}{36}\cdot\frac{a^n}{b^n}\frac{\sqrt{a^4b^4}}{a^{-3}b^{-4}} = \frac{8}{9}\left(\frac{a}{b}\right)^n \frac{a^2\frac{1}{b^2}}{\frac{1}{a^3}\cdot\frac{1}{b^4}} = \frac{8}{9}\left(\frac{a}{b}\right)^n a^5b^2$

b) $p(x) = -x^2+\sqrt{3}x-\frac{1}{4} = -\left(x^2 - 2\frac{\sqrt{3}}{2}x + \frac{3}{4} - \frac{3}{4} + \frac{1}{4}\right)$

$= -\left(\left(x-\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2}\right) = -\left(x-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2}$

$p(x)$ antar sitt maximum i $x=\frac{\sqrt{3}}{2}$ och $p\left(\frac{\sqrt{3}}{2}\right)=\frac{1}{2}$

c) $\begin{cases} 5x + 3y = 2 \\ 3x + 2y = 1 \end{cases} \iff \begin{cases} 10x + 6y = 4 \\ 9x + 6y = 3 \end{cases} \iff \begin{cases} 10x+6y=4 \\ x = 1 \end{cases}$

$\iff \begin{cases} 6y = -6 \\ x = 1 \end{cases} \iff \begin{cases} y = -1 \\ x = 1 \end{cases}$

2) $(x-1)^2+(y+1)^2=r^2$ sätt in $P=(0,\frac{1}{3})$

$(-1)^2+\left(\frac{1}{3}+1\right)^2=r^2 \iff r^2=(-1)^2+\left(\frac{4}{3}\right)^2 = 1+\frac{16}{9}=\frac{25}{9}$

så $r=\frac{5}{3}$

cirkelns ekvation blir då $(x-1)^2+(y+1)^2=\frac{25}{9}$

3) Skriv om till $2\cdot\frac{3^{2x}}{3}=1-\frac{3^x}{3}$ låt $t=3^x$

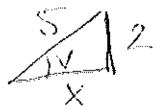
$2\cdot\frac{t^2}{3}=1-\frac{t}{3} \iff 2t^2=3-t \iff t^2+\frac{1}{2}t-\frac{3}{2}=0$

$t=-\frac{1}{4}\pm\sqrt{\frac{1}{16}+\frac{3}{2}} = -\frac{1}{4}\pm\sqrt{\frac{1}{16}+\frac{24}{16}} = -\frac{1}{4}\pm\frac{5}{4}$

$3^{x_1}=t_1=+1$ $x_1=0$

$3^{x_2}=t_2=-\frac{3}{2}$ $x_2=$ Går ej ty $3^x>0$ för alla reella x.

4a) $\sin(v)=-\frac{2}{5}$



$x^2+4=25$
$x^2=21$
$x=\sqrt{21}$

Tredje kvadranten ger $\cos(v)<0$, $\tan(v)>0$ så $\cos(v)=-\frac{\sqrt{21}}{5}$, $\tan(v)=\frac{2}{\sqrt{21}}$

b) $\cos(v)=\frac{1}{2} \Rightarrow v_{1,2}=\pm\frac{\pi}{3}+2\pi\cdot n$ där $n\in\mathbb{Z}$

4) $\cos(3v) = \cos\left(5v - \frac{\pi}{3}\right)$

$3v_1 = 5v_1 - \frac{\pi}{3} + 2\pi n \iff -2v_1 = -\frac{\pi}{3} + 2\pi n \iff v_1 = \frac{\pi}{6} + \pi n$

$3v_2 = -5v_2 + \frac{\pi}{3} + 2\pi n \iff 8v_2 = \frac{\pi}{3} + 2\pi n \iff v_2 = \frac{\pi}{24} + \frac{\pi n}{4}$

5) $\sum_{k=1}^{100} 3k+7 = 3\sum_{k=1}^{100} k + 7\sum_{k=1}^{100} 1 = 3\,\frac{100 \cdot 101}{2} + 7 \cdot 100$

$= 3 \cdot 101 \cdot 50 + 7 \cdot 100 = 3 \cdot 5050 + 700 = 15150 + 700 = \underline{15850}$

$K(x) = x^2 + 2x - 5$

6) $x^4 + 3x^3 - 9x^2 - 7x + 21 \;\big|\; \underline{x^2 + x - 6}$

$-(x^4 + x^3 - 6x^2)$

$2x^3 - 3x^2 - 7x + 21$

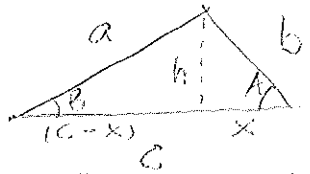
$-(2x^3 + 2x^2 - 12x)$

$-5x^2 + 5x + 21$

$-(-5x^2 - 5x + 30) = R(x)$

$10x - 9$

7) För triangeln med vinklar A, B, C och motstående sidor a, b resp. c gäller $a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$



1) $a^2 = h^2 + (c-x)^2 = h^2 + c^2 - 2cx + x^2$

2) $b^2 = h^2 + x^2$

Använd 2) i 1) och få

$a^2 = b^2 + c^2 - 2cx$

men $\frac{x}{b} = \cos(A)$ dvs $x = b\cos(A)$ så

$a^2 = b^2 + c^2 - 2bc\cos(A)$

QED.