Lösningar Introduktionskurs 2012-09-01 (MMG10)

1) a) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6+4+3}{12}=\underline{\underline{\frac{13}{12}}}$ (Svar)

b) $\begin{cases}3x+2y=1\\6x+5y=4\end{cases}\Leftrightarrow\begin{cases}3x+2y=1\\0x+y=2\end{cases}\Leftrightarrow\underline{\underline{\begin{cases}x=-1\\y=2\end{cases}}}$ (Svar)

c) $x^2+\frac{4}{3}x+\frac{5}{3}=1$

$x_{1,2}=-\frac{2}{3}\pm\sqrt{\frac{4}{9}-\frac{5}{3}}=-\frac{2}{3}\pm\sqrt{\frac{-11}{9}}=\underline{\underline{\frac{-2\pm i\sqrt{11}}{3}}}$ (Svar)

d) $\frac{a^{1/6}\cdot a^{-\frac{2}{3}}}{a^{1/4}\cdot a^{1/6}}=\frac{1}{a^{\left(\frac{1}{2}+\frac{2}{3}\right)}}=\frac{1}{a^{7/6}}=\underline{\underline{a^{-7/6}}}$ (Svar)

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2) a) $2\ln(3x+4)+2\ln 3=2\ln 2$

$\Leftrightarrow \ln(3x+4)=\ln\frac{2}{3}\Leftrightarrow 3x+4=\frac{2}{3}$

$\Leftrightarrow 3x=\frac{2}{3}-4=-\frac{10}{3}\Leftrightarrow \underline{\underline{x=-\frac{10}{9}}}$ (Svar)

b) $e^{2x}+\frac{3}{2}e^x-1=0\Leftrightarrow$

$e^x=-\frac{3}{4}\pm\sqrt{\frac{9}{16}+1}=-\frac{3}{4}+\frac{5}{4}=\begin{cases}\frac{1}{2}\\-2\end{cases}$

$x_1=\ln\frac{1}{2}=\underline{\underline{-\ln 2}}$ (Svar) (x_2 ej reell)

3a) $x^2+\frac{2}{3}x+y^2-\frac{4}{3}y+\frac{1}{3}=0\Leftrightarrow$

$x^2+\frac{2}{3}x+\frac{1}{9}+y^2-\frac{4}{3}y+\frac{4}{9}-\frac{1}{9}-\frac{4}{9}+\frac{1}{3}=0$

$\Leftrightarrow\left(x+\frac{1}{3}\right)^2+\left(y-\frac{2}{3}\right)^2=\frac{2}{9}$ (Svar:) $(x_0,y_0)=\left(-\frac{1}{3},\frac{2}{3}\right)$, $\underline{\underline{R=\frac{\sqrt{2}}{3}}}$

3b) $2x^3+2x^2+4x+5 \;|\; x^2-3x+1$

$-(2x^3-6x^2+2x) \;|\; 2x+9 \equiv K(x)$

$8x^2+2x+5$

$-(8x^2-27x+9)$

$29x-4 \equiv R(x)$

(Svar:)

4) Bilda $K(x) = \frac{2x+3}{3x+2} - \frac{x+3}{x+2} \equiv \frac{(2x+3)(x+2)-(x+3)(3x+2)}{(3x+2)(x+2)} \equiv \frac{(2x^2+7x+6)-(3x^2+11x+6)}{(3x+2)(x+2)}$

$\equiv \frac{-x^2-4x}{(3x+2)(x+2)} \equiv \frac{-x(x+4)}{(3x+2)(x+2)} \equiv \frac{-x(x+4)}{3\left(x+\frac{2}{3}\right)(x+2)} < 0$

Teckenstudium:

x		-4		-2		$-\frac{2}{3}$		0	
$K(x)$	$-$	$\mid$	$+$	$\mid$	$-$	$\mid$	$+$	$\mid$	$-$

Svar: $x>0,\ -2<x<-\frac{2}{3},\ x<-4$

5) $\cos(2x)+3\cos x+2=0 \Leftrightarrow 2\cos^2 x-1+3\cos x+2=0$

$\Leftrightarrow \cos^2 x+\frac{3}{2}\cos x+\frac{1}{2}=0$

$\Leftrightarrow \cos x = -\frac{3}{4} \pm \sqrt{\frac{9}{16}-\frac{1}{2}} = -\frac{3}{4}\pm\frac{1}{4} = \begin{cases} -\frac{1}{2} \\ -1 \end{cases}$

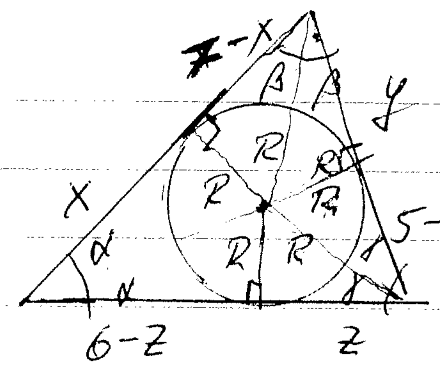
Fall 1: $\cos x = -\frac{1}{2}$ $(0^\circ < x < 360^\circ)$

$x = 150^\circ + n\cdot 360^\circ$ eller $210^\circ + n\cdot 360^\circ \Rightarrow 150^\circ, 210^\circ$

Fall 2 $\cos x = -1$ $(0^\circ < x < 360^\circ) \Rightarrow x = 180^\circ + n\cdot 360^\circ \Rightarrow 180^\circ$

Svar: $150^\circ, 180^\circ, 210^\circ$

6)



(Antag: Se figur)

Geometrin ger
$$\begin{cases} x = 6 - z \\ y = 7 - x \\ z = 5 - y \end{cases} \iff \begin{cases} x = 4 \\ y = 3 \\ z = 2 \end{cases}$$

Vi har $2\alpha + 2\beta + 2\gamma = 180^\circ \iff \alpha + \beta + \gamma = 90^\circ$

och $\tan\alpha = \frac{R}{4}$, $\tan\beta = \frac{R}{3}$, $\tan\gamma = \frac{R}{2}$

$\therefore \tan(\alpha+\beta) = \tan(90^\circ - \gamma) = \cot\gamma = \frac{1}{\tan\gamma}$

där $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$$\therefore \frac{\frac{R}{4} + \frac{R}{3}}{1 - \frac{R}{4}\cdot\frac{R}{3}} = \frac{2}{R}$$

$$\iff \frac{7R^2}{12 - R^2} = 2 \iff 7R^2 = 24 - 2R^2$$

$$\iff R^2 = \frac{8}{3} \iff R = \frac{2\sqrt{2}}{\sqrt{3}}$$

Svar: $R = 2\sqrt{\frac{2}{3}}$

/RP