MMG 300 del 2
11-08-22

(1) Sats 9.4 ; P.B.

(2) Sats 2.8 ; GLO

(3) Sats 2.12 ; GLO

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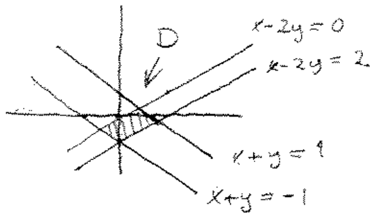
(4) Sätt $\begin{cases} u = x-2y \\ v = x+y \end{cases}$ $D'$: $\begin{matrix} 0<u<2 \\ -1<v<1 \end{matrix}$




$\frac{d(u,v)}{d(x,y)} = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = 3$

$\iint_D y\,dxdy = \frac{1}{3}\iint_{D'} \frac{1}{3}(v-u)\,dudv =$

$= \frac{1}{9}\int_0^2 \left[\frac{v^2}{2} - uv\right]_{v=-1}^{1} du = -\frac{2}{9}\int_0^2 u\,du = \underline{-\frac{4}{9}}$

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(5) $r_u' \times r_v' = (2v, 2u, 0) \times (2u, 0, 4v) = \cancel{4u(2v, 2u, ...)}$ $4(2uv, -2v^2, -u^2)$

$A = \int_1^2 \left(\int_1^3 |r_u' \times r_v'|\,du\right) dv = \int_1^2 \left(\int_1^3 4\sqrt{u^4 + 4v^4 + 4u^2v^2}\,du\right) dv =$

$= 4\int_1^2 \left(\int_1^3 (u^2 + 2v^2)\,du\right) dv = 8\int_1^2 u^2 du + 8\int_1^3 v^2 dv =$

$= 8\left(\left(\frac{8}{3} - \frac{1}{3}\right) + \left(\frac{27}{3} - \frac{1}{3}\right)\right) = \underline{88}$

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(6) $|a_k|^{1/k} = \frac{(k+1)^{1/k} \cdot 2|z|^2}{\left[k(\ln k)(\ln\ln k)^2\right]^{1/k}} \to 2|z|^2$

eftersom nämnaren för stora k är instängd mellan $k^{1/k}$ och $(2k)^{1/k}$ som båda $\to 1$, Dvs $\underline{R = \frac{1}{\sqrt{2}}}$

För $|z| = \frac{1}{\sqrt{2}}$ är $|a_k| \le \frac{1}{k \ln k (\ln\ln k)^2}$ och

$\int_3^\infty \frac{1}{x \ln x (\ln\ln x)^2}\,dx = \left[\begin{matrix} t = \ln\ln x \\ dt = \frac{1}{x\ln x} \end{matrix}\right] = \int_{\ln\ln 3}^\infty \frac{1}{t^2}\,dt$ som är

konvergent. Integralkreteriet ger sen påståendet

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(7) $\mathrm{Div}(xy, -2y, z-x) = y-1.$

Gauss' sats ger därför integralen

$\iiint_D (y-1)\,dxdydz = \iint_{x^2+y^2\le 4} \left(\int_{x^2+y^2}^{4} (y-1)\,dz\right) dxdy =$

$= [\text{polära koordinater}] = \int_0^2 \left(\int_0^{2\pi} (r\sin\theta - 1)(4-r^2)\,r\,d\theta\right) dr$

$= -2\pi\int_0^2 (4r - r^3)\,dr = -2\pi\left[2r^2 - \frac{r^4}{4}\right]_0^2 = \underline{-8\pi}$

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(8) $y = \sum_0^\infty c_k x^k$

$y' = \sum_1^\infty c_k k x^{k-1} = \sum_0^\infty c_{k+1}(k+1)x^k$

$y'' = \sum_0^\infty c_{k+1} k(k+1) x^{k-1}$

$xy'' = \sum_0^\infty c_{k+1} k(k+1) x^k$

Insättning i ekvation ger

$\sum_0^\infty \left(c_{k+1}k(k+1) + c_{k+1}(k+1) - c_k\right) x^k = 0$

Dvs $c_{k+1}[k(k+1) + k+1] = c_k$ eller

$c_{k+1} = \frac{1}{(k+1)^2} c_k$

$y(0) = 1 \Rightarrow c_0 = 1$ och

$c_1 = \frac{1}{1^2}c_0 = 1$, $c_2 = \frac{1}{2^2}$, $c_3 = \frac{1}{3^2}\cdot\frac{1}{2^2}, \ldots$

$\ldots c_k = \frac{1}{k^2(k-1)^2 \cdots 2^2 \cdot 1} = \frac{1}{(k!)^2}$

$\underline{y = \sum_0^\infty \frac{x^k}{(k!)^2}}$