Allmänna Stokes' sats

Differentialformer av olika ordning i R^n;
k-former, ω, $k = 0, 1, \ldots, n$.

n	k	ω
1	0	$f(x)$
	1	$f(x)dx$
2	0	$f(x,y)$
	1	$f dx + g dy$
	2	$f dx dy$
3	0	$f(x,y,z)$
	1	$f dx + g dy + h dz$
	2	$f dx dy + g dx dz + h dy dz$
	3	$f dx dy dz$
4	0	$f(x_1, x_2, x_3, x_4)$
	1	$f_1 dx_1 + f_2 dx_2 + f_3 dx_3 + f_4 dx_4$
	2	$f_1 dx_1 dx_2 + f_2 dx_1 dx_3 + f_3 dx_1 dx_4 + f_4 dx_2 dx_3 + f_5 dx_2 dx_4 + f_6 dx_3 dx_4$
	3	$f_1 dx_1 dx_2 dx_3 + f_2 dx_1 dx_2 dx_4 + f_3 dx_1 dx_3 dx_4 + f_4 dx_2 dx_3 dx_4$
	4	$f dx_1 dx_2 dx_3 dx_4$

k-former integreras över k-dimensionella mängder (k-parametriga mängder)

Om $k = n$ (i R^n) så blir det
enkel-, dubbel-, trippel-, multipel-integraler

Differentiering av k-former

Ex 1) $\omega = f(x,y)$, 0-form i R^2:

$d\omega = f'_x\,dx + f'_y\,dy$ 1-form i R^2

2) $\omega = f\,dx_1\,dx_3$, 2-form i R^4

$d\omega = (f'_{x_1}dx_1 + f'_{x_2}dx_2 + f'_{x_3}dx_3 + f'_{x_4}dx_4)\,dx_1\,dx_3 =$

$= (\,dx_j\,dx_j = 0$, lika index$) =$

$f'_{x_2}\,dx_2\,dx_1\,dx_3 + f'_{x_4}\,dx_4\,dx_1\,dx_3 =$

$=$ (sortering: platsbyte ger teckenbyte) $=$

$= -f'_{x_2}\,dx_1\,dx_2\,dx_3 + f'_{x_4}\,dx_1\,dx_3\,dx_4$

en 3-form i R^4

Stokes' sats:
(Allmänna)

$$\boxed{\int_{\partial D} \omega = \int_D d\omega}$$

ω: k-form

$0 \le k \le n-1$

$\dim D \in [1,\dots,n]$
$=k+1$

$n=3,\ k=2$ ger Gauss' sats

$n=3,\ k=1$ ger Stokes' sats
(klassiska)

$n=2,\ k=1$ ger Greens formel

$n=1,\ k=0$ ger $f(b)-f(a) = \int_a^b f'(x)\,dx$

$n=2,\ k=0$ ger $U(B)-U(A) = \int_\gamma \underbrace{U'_x dx + U'_y dy}_{\text{exakt.}}$

γ: A till B

$$\underline{d(Pdx + Qdy)} =$$

$$= (\cancel{P'_x dx} + P'_y dy)dx + (Q'_x dx + \cancel{Q'_y dy})dy =$$

$$= P'_y\, dydx + Q'_x\, dxdy = \underline{(Q'_x - P'_y)\, dxdy}$$

$$\underline{d(Pdx + Qdy + Rdz)} =$$

$$= (\cancel{P'_x dx} + P'_y dy + P'_z dz)\, dx +$$

$$+ (Q'_x dx + \cancel{Q'_y dy} + Q'_z dz)\, dy +$$

$$+ (R'_x dx + R'_y dy + \cancel{R'_z dz})\, dz =$$

$$P'_y\, dydx + P'_z\, dzdx +$$

$$+ Q'_x\, dxdy + Q'_z\, dzdy +$$

$$+ R'_x\, dxdz + R'_y\, dydz =$$

$$= \underline{(R'_y - Q'_z)\, dydz + (P'_z - R'_x)\, dzdx + (Q'_x - P'_y)\, dxdy}$$

$$\underline{d(Pdydz + Qdzdx + Rdxdy)} =$$

$$(P'_x dx + \cancel{P'_y dy} + \cancel{P'_z dz})\, dydz +$$

$$+ (\cancel{Q'_x dx} + Q'_y dy + \cancel{Q'_z dz})\, dzdx +$$

$$+ (\cancel{R'_x dx} + \cancel{R'_y dy} + R'_z dz)\, dxdy =$$

$$= \underline{(P'_x + Q'_y + R'_z)\, dxdydz}$$