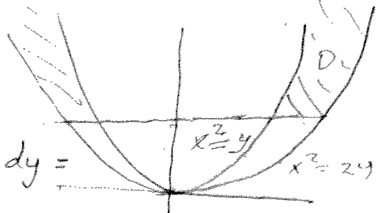
Lösningar till
MMG 300-2
12-05-31

① Sats 3.3 i GLO

② Sats 9.3 i P-B.

③ Sats 4 6.4 i P B.



④ $\iint_D \frac{1}{x^2 y}\,dx\,dy = \{\text{Symmetri}\} = 2\int_0^\infty \frac{1}{y}\left(\int_{\sqrt{y}}^{\sqrt{2y}} \frac{1}{x^2}\,dx\right)dy =$

$= 2\int_1^\infty \frac{1}{y}\left[-\frac{1}{x}\right]_{\sqrt{y}}^{\sqrt{2y}} dy = 2\int_1^\infty \frac{1}{y}\left(\frac{1}{\sqrt{y}} - \frac{1}{\sqrt{2y}}\right)dy = (2-\sqrt{2})\int_1^\infty \frac{1}{y\sqrt{y}}\,dy =$

$= (2-\sqrt{2})\left[-\frac{2}{\sqrt{y}}\right]_1^\infty = \underline{\underline{4-2\sqrt{2}}}$

⑤ Greens formel ger

$\int_C (x+y^3)\,dx - (y+x^3)\,dy =$

$= \iint_D \frac{\partial}{\partial x}(-y-x^3) - \frac{\partial}{\partial y}(x+y^3)\,dx\,dy =$

$= \iint_D -3(x^2+y^2)\,dx\,dy = [\text{polära kordinater}] =$

$-3\int_{\pi/4}^{3\pi/4}\left(\int_0^1 r^2\cdot r\,dr\right)d\theta = -\frac{3\pi}{2}\left[\frac{r^4}{4}\right]_0^1 = \underline{\underline{-\frac{3\pi}{8}}}$

⑥ Konvergensradie?

Eftersom $\frac{2^n}{2n^2} \le \frac{1+\sqrt{n}\,2^n}{n+n^2} \le 2\sqrt{n}\,2^n$ och

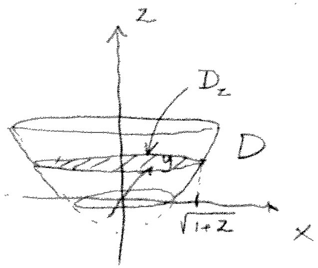
$\left|\frac{2^n}{2n^2} z^{2n}\right|^{1/n} \to 2|z|^2$ liksom $\left|2\sqrt{n}\,2^n z^{2n}\right|^{1/n} \to 2|z|^2$

Så konvergera serien för $2|z|^2 < 1$, dvs $R = \frac{1}{\sqrt{2}}$.

Om $|z| = \frac{1}{\sqrt{2}}$ konstaterar vi att $\sum_1^\infty \left|\frac{2^{-n}+\sqrt{n}}{n+n^2} z^{2n}\right| =$

$\sum_1^\infty \frac{2^{-n}+\sqrt{n}}{n+n^2}$ som är likakonvergent som $\sum_1^\infty \frac{1}{n^{3/2}}$ eftersom

$\frac{2^{-n}+\sqrt{n}}{n+n^2} \Big/ \frac{1}{n^{3/2}} \to 1$ då $n \to \infty$. Svar: konvegent för $\underline{\underline{|z| \le \frac{1}{\sqrt{2}}}}$



⑦ $\iiint_D (x+y^2+z^3)\,dx\,dy\,dz =$

$= \int_0^1 \left(\iint_{D_z} (x+y^2+z^3)\,dz\right) =$

$= \begin{bmatrix}\text{polära}\\ \text{koord. i}\\ \text{xy-led}\end{bmatrix} = \int_0^1\left(\int_0^{\sqrt{1+z}}\left(\int_0^{2\pi} r^2\cos\theta + r^3\sin^2\theta + r z^3\,d\theta\right)dr\right)dz =$

$= \int_0^1\left(\int_0^{\sqrt{1+z}}\left[r^2\sin\theta + r^3\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right) + r z^3\theta\right]_0^{2\pi} dr\right)dz =$

$= \int_0^1\left(\int_0^{\sqrt{1+z}} \pi r^3 + 2\pi r z^3\right)dr\,dz = \int_0^1 \left[\frac{\pi}{4} r^4 + \pi r^2 z^3\right]_{r=0}^{\sqrt{1+z}} dz =$

$= \pi \int \frac{(1+z)^2}{4} + (1+z)z^3\,dz = \pi\left[\frac{(1+z)^3}{12} + \frac{z^4}{4} + \frac{z^5}{5}\right]_0^1 = \underline{\underline{\frac{31\pi}{30}}}$

⑧ $\frac{x^3+n^2x}{(x+n)^2} = x\,\frac{\frac{x^2}{n^2}+1}{\left(\frac{x}{n}+1\right)^2} \to x$ då $x \to \infty$ punktvis.

Likformigt i $[0,1]$?

$x - \frac{x^3+n^2x}{(x+n)^2} = x\,\frac{(x+n)^2 - x^2 - n^2}{(x+n)^2} = \frac{2nx^2}{(x+n)^2} \le \frac{2n}{n^2} = \frac{2}{n} \to 0$

då $n \to \infty$, oberoende av x. Det visar

att konvergensen är likformig och att

därmed $\lim_{n\to\infty} \int_0^1 \frac{x^3+nx^2}{(x+n)^2}\,dx = \int_0^1 \lim_{x\to\infty} \frac{x^3+nx^2}{(x+n)^2}\,dx =$

$= \int_0^1 x\,dx = \underline{\underline{\frac{1}{2}}}$.