



CHALMERS UNIVERSITY OF TECHNOLOGY
MATHEMATICAL DEPARTMENT
BIOMATHEMATICS-TMV090
PROJECT
LIGHT BULBS-HEAT AND LIFE-TIME

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Abstract

The heat equation is very important in our life, because it describes the phenomenon of heat transfer throughout bodies, which is essential if we are to know how much heat that is transferred, and the temperature of the bodies.

Conduction is heat transfer by means of molecular agitation within a material without any motion of the material as a whole. In this project we investigate conduction of heat in different parts of a lamp and the gas inside it. Also we derive a model for the heat of an electrical bulb, which especially describes the heat of the glass surrounding the filament inside. In addition we give some knowledge on the mechanism of lighting, the types of electrical bulbs, the constituents of incandescent bulbs and life time of the bulbs.

Acknowledgment:

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1.0. Introduction

In real life there are many types of bulbs, the most important are the incandescent bulb and the fluorescent bulb, both used widely in industry, as light in our homes, streets, etc. To study such an object is very interesting on one side, and very important on the other.

Why is it interesting? To know how the electric bulbs work, how they produce light and from what they are made.

And why is it important? To study what really happens inside the bulb and why the glass of the bulb becomes very hot. Besides it is important to study the effect of heat on the life time of the bulb. All these questions and others are subjects of discussion for this project.

2.0. Electric bulb

Light bulb, like lots of devices has many types, but the most commonly used is the incandescent light, which generates more heat. Here we base our study on incandescent light, and some information about light bulbs is explained, i.e. main types of bulbs, history, basic parts, building up, and mechanism of incandescent light bulb.

2.1. Types of electrical bulbs

There are many types of electric bulbs that are used in real life, but the most used are incandescent light bulbs, fluorescent lamps, and gas discharge lamps. We give brief information about each one.

2.1.1. Incandescent light bulb

An incandescent light bulb produces light by heating a small filament of tungsten to about 1000-2500 °C. At that temperature, the thermal radiation that the filament emits includes a substantial amount of visible light. But the filament also emits a great deal of infrared light (heat light) and it also transfers heat via conduction and convection to the glass bulb around it. When you put your hand near the bulb, you feel both the infrared light and the heat that has worked its way to the surface of the bulb. The bulb feels hot. The efficiency of this type of electrical bulb is 5%, this means that approximately 95% of the energy is converted to heat and the remaining quantity is only converted to light. And we will apply our research to this kind of bulb since it produces much more heat than the others.

2.1.2. Fluorescent light bulb

A fluorescent lamp tries to produce light without heat. It is colliding electrons with mercury atoms to produce an atomic emission of ultraviolet light. This ultraviolet light is then converted to visible light by the layer of white phosphor powder on the inside of the lamp's glass envelope. In principle, this

whole activity can be performed without creating any thermal energy. However, many unavoidable imperfections cause the lamp to convert some of the electric energy it consumes into thermal energy. Nonetheless, the lamp only becomes warm rather than hot. And the efficiency of this type is 10%, it means that only 8.1% from the energy is converted to light.

2.1.3. Gas discharge lamp

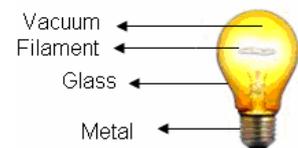
This lamp uses an electric discharge in a gas inside the lamp to produce visible light directly. (Often high pressure mercury, sodium vapor, or even neon).

2.2. History of the light bulb

The invention of the light bulb is usually attributed in Britain to Joseph Wilson Swan in 1828-1914, and in the United States to Thomas Alva Edison. Some sources indicate that Heinrich Göbel built the first functional bulbs three decades earlier. He later challenged Edison's patent while living in the United States, but his legal "interference case" was overruled in court. Alexander Nikolayevich Lodygin independently developed an incandescent light bulb in 1874

2.3. Basic parts of the incandescent bulb

The incandescent light bulb is made from a thin filament of a material with a high melting point sealed inside a glass bulb, from which the air has been evacuated or which is filled with an inert gas to protect the filament from oxidation (i.e. to prevent the filament from burning out at high temperature).



The filament is made from a material with high melting points and high resistance, because the proportion of light energy to heat energy radiated by the filament rises as the temperature increases, and the most efficient light source is obtained at the highest filament temperature, and the needs of high resistance is to give more light, so almost every filament is made from tungsten, because it has these properties.

The burning of the lamp cause the filament to become thinner during its life to the point where it breaks at the thinnest point, this point well melt just before failure.

2.4. Building up light bulb

The glass enclosures are made from a ribbon of hot glass that's first thickened and then blown into molds to form the bulb shapes. These enclosures are then cooled, cut from the ribbon, and their insides are coated with diffusing material that gives the finished bulb its soft white appearance.

The filament is formed by drawing tungsten metal into a very fine wire. This wire, typically only 42 microns (0.0017 inches) in diameter is first wound into a coil and then this coil is itself wound into a coil. The mandrels used in these two coiling processes are trapped in the coils and must be dissolved

away with acids after the filament has been annealed.

The finished filament is clamped or welded to the power leads, which have already been embedded in a glass supporting structure. This glass support is inserted into a bulb and the two glass parts are fused together. A tube in the glass support allows the manufacturer to pump the air out of the bulb and then reintroduce various inert gases. When virtually all of the oxygen has been eliminated from the bulb, the tube is cut off and the opening is sealed. Once the base of the bulb has been attached, the bulb is ready for use.

2.5. Mechanism of light production in the bulb

When the electrical current travels through the filament and because of the electrical resistance of the filament, the filament gets white-hot and generates light. In detail, an electric current flows through a double-spiral coil of very thin tungsten wire. As the electric charges in the current flow through this tungsten filament, they collide periodically with the tungsten atoms and transfer energy to those tungsten atoms. The current gives up its energy to the tungsten filament and the filament's temperature rises to about 1000°C to 2500°C . While all objects emit thermal radiation, very hot objects emit some of the thermal radiation as visible light. An object of 1000C° to 2500C° emits about 12% of its heat as visible light and this is the light that you see coming from the bulb. The glass enclosure shields the filament from oxygen because tungsten burns in air.

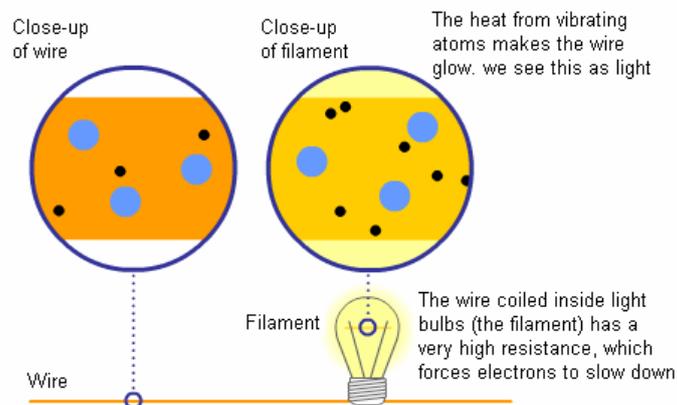


Figure 1: The electric motion through the conductors

Electrons in copper wire have an easy time breaking free from the pull of their atoms, therefore copper is said to have low resistance. The easier it is for electrons to break free, the better electricity flows, and the lower resistance a material has.

But materials with high resistance usually have fewer electrons available to break away, and often the atoms are closer together. This means a stronger pull on each electron, which makes them slow down. Because the electrons keep fighting to escape, the atoms shake back and forth and heat up. In light bulbs, we see the glow from that heat as light.

3.0. Modeling of the system

Derivation of the mathematical model are of the most fundamental principles of physics, it depends on the law of conservation of energy. According to this principle energy can't be created or destroyed, but transformed from one form to another, or transfer from one source to another.

In this problem the heat transfer is proportional to two types of heat transfer, conduction and convection.

3.1. Conduction heat transfer

First we derive the heat *conduction* through arbitrary volume Ω in R^2 with boundary Γ over a time $I = [0, T]$. We let $u(x, t)$ denote the temperature and $q(x, t)$ the heat at the point x at time t . We let $f(x, t)$ denote the rate of heat (per unit of volume) supplied at (x, t) by a heat source.

We derive the model using a basic *conservation law* expressing conservation of heat in the following form: for any fixed domain V in Ω with boundary S , the rate of the total heat introduced in V by the external source is equal to the rate of the total heat accumulated in V plus the total heat flux through S . This is based on the fact that the convection that the heat introduced in V by the external source can choose between two options only: (1) flow out of V or (2) be accumulated in V . With S denoting the boundary of V and n denoting the outward unit normal to S , the conservation law can be expressed as

$$\int_V f \, dx = \frac{\partial}{\partial t} \int_V \lambda u \, dx + \int_S q \cdot n \, ds, \quad (1)$$

Where $\lambda(x, t)$ is the heat capacity coefficient that gives the amount of heat per unit of volume needed to raise the temperature one unit, and for the glass it equals $0.84 \, \text{kJ} / \text{kg} \cdot \text{K}$, and all functions are evaluated at a specific time, $t \in I$. By the divergence theorem,

$$\int_S q \cdot n \, ds = \int_V \nabla \cdot q \, dx, \quad (2)$$

And combined with (1), this implies that:

$$\int_V \left(\frac{\partial}{\partial t} (\lambda u) + \nabla \cdot q \right) dx = \int_V f \, dx, \quad (3)$$

Where the time derivation could be moved under the integral sign because V does not depend on the time t . Since V is arbitrary, assuming the integrands are Lipschitz continuous, it follows that

$$\frac{\partial}{\partial t} (\lambda u)(x, t) + \nabla \cdot q(x, t) = f(x, t) \quad \text{for all } x \in \Omega, 0 < t \leq T \quad (4)$$

That is a differential equation describing conservation of heat involving two unknown: the temperature $u(x, t)$ and the heat flux $q(x, t)$. We thus have one equation and two unknowns and we need yet

another equation.

The second equation is a *constitutive equation* that couples the heat flux q to the temperature gradient ∇u . Fourier's law states that heat flows from warm to cold regions with the heat flux proportional to the temperature gradient:

$$q(x,t) = -a(x,t)\nabla u(x,t) \quad \text{for } x \in \Omega, \quad 0 < t \leq T \quad (5)$$

Where the factor of proportionality $a(x,t)$ is the coefficient of heat conductivity, and for glass it equals 0.5 W/m.K . Note the minus sign indicating that the heat flows from warm to cold regions, and that the heat conductivity $a(x,t)$ is positive. Combining (4) and (5) we obtain the basic differential equation describing heat conduction:

$$\frac{\partial}{\partial t}(\lambda u) - \nabla \cdot (a \nabla u) = f \quad \text{in } \Omega \times (0, T], \quad (6)$$

Where $a(x,t)$ and $\lambda(x,t)$ are given positive coefficients depending on (x,t) and $f(x,t)$ is a given heat source, and the unknown $u(x,t)$ represents the temperature.

Provided the differential equation is complemented by initial and boundary conditions. The complete model with Dirichlet boundary conditions reads:

$$\begin{cases} \frac{\partial}{\partial t}(\lambda u) - \nabla \cdot (a \nabla u) = f & \text{in } \Omega \times (0, T], \\ u = u_b & \text{on } \Gamma \times (0, T], \\ u(x,0) = u_0(x) & \text{for } x \in \Omega, \end{cases} \quad (7)$$

And the stationary heat conduction equation is:

$$\begin{cases} -\nabla \cdot (a \nabla u) = f & \text{in } \Omega \times (0, T], \\ u = u_b & \text{on } \Gamma \times (0, T], \end{cases} \quad (8)$$

3.2. Convection heat transfer

Now we will formulate the heat transfer by *convection*, if we assume that the heat transfer is convection dominated, corresponding to the conservation of heat for any fixed domain V in Ω with boundary S , the rate of the total heat introduced in V by the external source is equal to the rate of the total heat accumulated in V plus the total heat flux through S by the convection

$$\int_V f \, dx = \frac{\partial}{\partial t} \int_V \lambda u \, dx + \int_S q \cdot n \, ds \quad (9)$$

And by the divergence theorem

$$\int_S q \cdot n \, ds = \int_V \nabla \cdot q \, dx, \quad (10)$$

Substitute 10 in 9 we get

$$\int_V \left(\frac{\partial}{\partial t} (\lambda u) + \nabla \cdot q \right) dx = \int_V f dx, \quad (11)$$

And this gives

$$\frac{\partial}{\partial t} (\lambda u)(x, t) + \nabla \cdot q(x, t) = f(x, t) \quad \text{for all } x \in \Omega, 0 < t \leq T \quad (12)$$

Where

$$\nabla \cdot q = \beta \cdot \nabla u, \quad (13)$$

Here β is the vector field velocity,

Combining equation (12) and (13),

This give the final form of heat transfer by convection

$$\begin{cases} \frac{\partial}{\partial t} (\lambda u) + \beta \cdot \nabla u = f & \text{in } \Omega \times (0, T], \\ u = u_b & \text{on } \Gamma \times (0, T], \\ u(x, 0) = u_0(x) & \text{for } x \in \Omega, \end{cases} \quad (14)$$

Here we consider the boundary condition as Dirichlet boundary conditions just as an example. And the stationary case corresponding to this equation is

$$\begin{cases} \beta \cdot \nabla u = f & \text{in } \Omega \times (0, T], \\ u = u_b & \text{on } \Gamma \times (0, T], \end{cases} \quad (15)$$

So we can formulate the model of the glass heat equation as follows:

$$\begin{cases} \frac{\partial}{\partial t} (\lambda u) - \nabla \cdot (a \nabla u) + \beta \cdot \nabla u = f & \text{in } \Omega \times (0, T], \\ u = u_b & \text{on } \Gamma \times (0, T], \\ u(x, 0) = u_0(x) & \text{for } x \in \Omega, \end{cases} \quad (16)$$

With the source given by $f(x, t)$, and given boundary conditions and initial boundary

4.0. Modeling of light bulb

4.1. Geometry 3D - 2D

In the case of the light bulb we have a 3D geometric body, in fact the problem is three dimensions 3D (see fig 2), and because of the symmetry of the light bulb at the z - axis, we will deal with the problem as a two dimension problem (2D), as shown in figure 3.

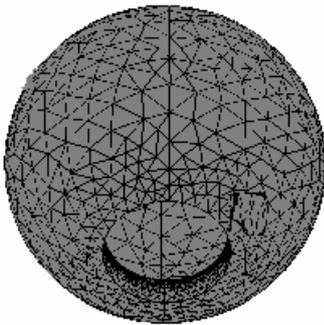


Figure 2: 3D Geometry of the bulb

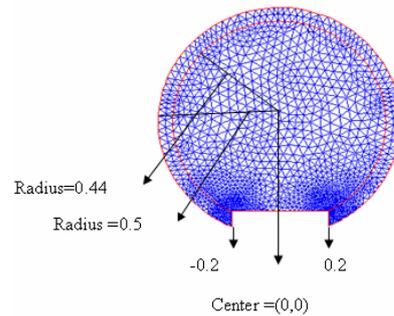


Figure 3: 2D Geometry of the bulb

The exact model is formulated for both the glass and the gas inside the bulb; here the heat conduction coefficient is changing from the gas medium to the coefficient of glass, so we will define two coefficients. Let a_1 be the conduction coefficient of the gas, and a_2 be the conduction coefficient of the glass. The gas inside the bulb will occupy a circle with radius $r_2 = 0.44$, and the thickness of the glass which is taken as 0.06 unit length ($r_1 - r_2 = 0.5 - 0.44 = 0.06$), also we have the metal in the bottom of the light bulb, this metal is of high conduction coefficient, which means it will have high temperature, and it is formulated to occupy a cylinder with radius 0.2 in 3D, and in 2D it is formulated as the boundary at two points -0.2 and 0.2.

4.2. Simplified model

Reasonable consideration must be taken because the convection field is too small to be mentioned, since the bulb is evacuated from air, also the convection term will be too small compared to the conduction terms, so we will let $\beta = 0$.

In fact the glass will be rapidly heated and we can consider the steady state case, and the problem will be time independent. The problem becomes to find the temperature of the surface of the glass with thickness 0.06 that is having a Nobel gas inside, in other words to find the temperature $u(x,t)$ such that

$$\begin{cases} -\nabla \cdot (a\nabla u) = f & \text{in } \Omega, \\ u = u_b & \text{on } \Gamma, \end{cases} \quad (17)$$

4.3. The source function $f(x, y)$

Now $f(x, y)$ is considered as the heat the filament generates, which means that:

$f(x, y) = (1 - \text{efficiency of the bulb}) \times \text{the bulb energy} = 0.95 \times \text{the bulb energy}$, and this is equal to the temperature of the tungsten $1700C^\circ$ in the rate, and since that of the filament is very small inside the bulb, and also because the heat is uniformly distributed at the surface of the filament, we will consider the filament as a small sphere centered at the centre of the bulb, so in the $2D$ -case we will take the source as a small circle centered at the origin of the geometry.

$$f(x, y) = 1700 \text{ whenever } \text{norm}(x, y) \leq 0.1 \text{ (the average of 1000 and 2500)}$$

4.4. Thermal conductivity coefficient

The thermal conductivity coefficients are different from gas space to the glass, so we can define a as follows:

$$\begin{cases} a_1, & x^2 + y^2 < 0.44^2, & \text{Gas - space} \\ a_2, & 0.44^2 \leq x^2 + y^2 \leq 0.5^2, & \text{Glass - space} \end{cases} \quad (18)$$

Where $a_1 = 0.0314$, and $a_2 = 0.35$.

$$\begin{cases} 0.0314, & x^2 + y^2 < 0.44^2, \\ 1, & 0.4^2 \leq x^2 + y^2 \leq 0.5^2, \end{cases} \quad (19)$$

5.0. Boundary conditions

The boundary condition considered consists of two types of boundary condition, Dirichlet boundary condition in Γ_1 (green color), and Neumann boundary condition Γ_2 and Γ_3 (red and yellow color respectively), this is depicted in fig 4.

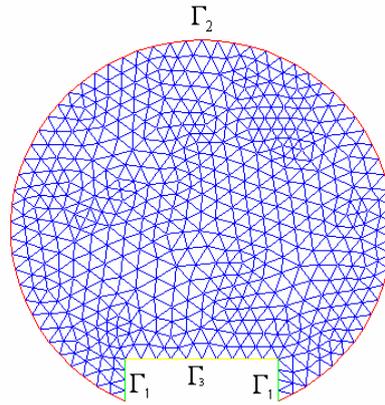


Figure 4: The boundary of the system

This is reasonable, since there is an exchange of temperature between glass at Γ_2, Γ_3 and the surrounding air, whereas aluminum at Γ_1 is isolated from the outside by the plug.

5.1. boundary Γ_1

At boundary Γ_1 , the material that is in touch with the glass is metal and is made from aluminum, and this metal is of high conductivity that is heated quickly, so we can consider the temperature of the glass at this boundary as the temperature of the metal. In fact the aluminum is almost only heated from the filament, but a small part of its temperature is coming from the flow of the electric current that is passing through it. As shown in the following figure (5), we can formulate the problem for the whole system, air, gas glass, and aluminum (AGGA) inside the bulb.

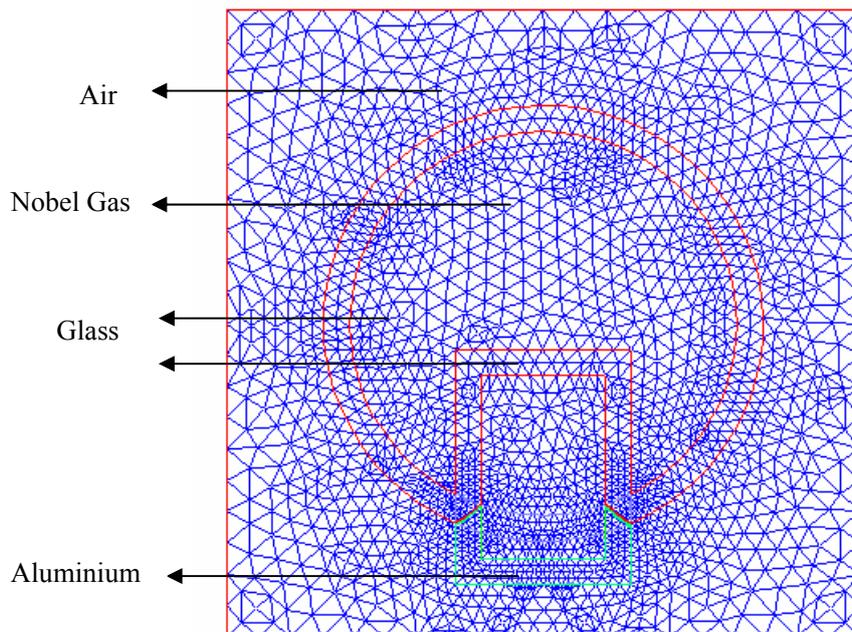


Figure 5: The whole system, Air, Gas, Glass, Aluminium (AGGA)

So to determine the temperature at this boundary it is better to define the temperature at the boundary Γ_1 as the sum of two temperature values, the temperature from the filament and the temperature from current flow.

5.1.1. Temperature gained from the filament

This value can be obtained by solving the whole system, air, glass, gas, and aluminum (AGGA), the mesh will be like the figure depicted above, and the method that is used to get the solution is the Finite Element Method (FEM) for solving partial differential equations (PDE), we will put the light on this method later. After solving the system with different conduction coefficients $a_1 = 0.0314$, $a_2 = 1$, $a_3 = 240$, $a_4 = 0.0257$ for gas, glass, aluminum, and air respectively, then the solution of the system is depicted in the following figure:

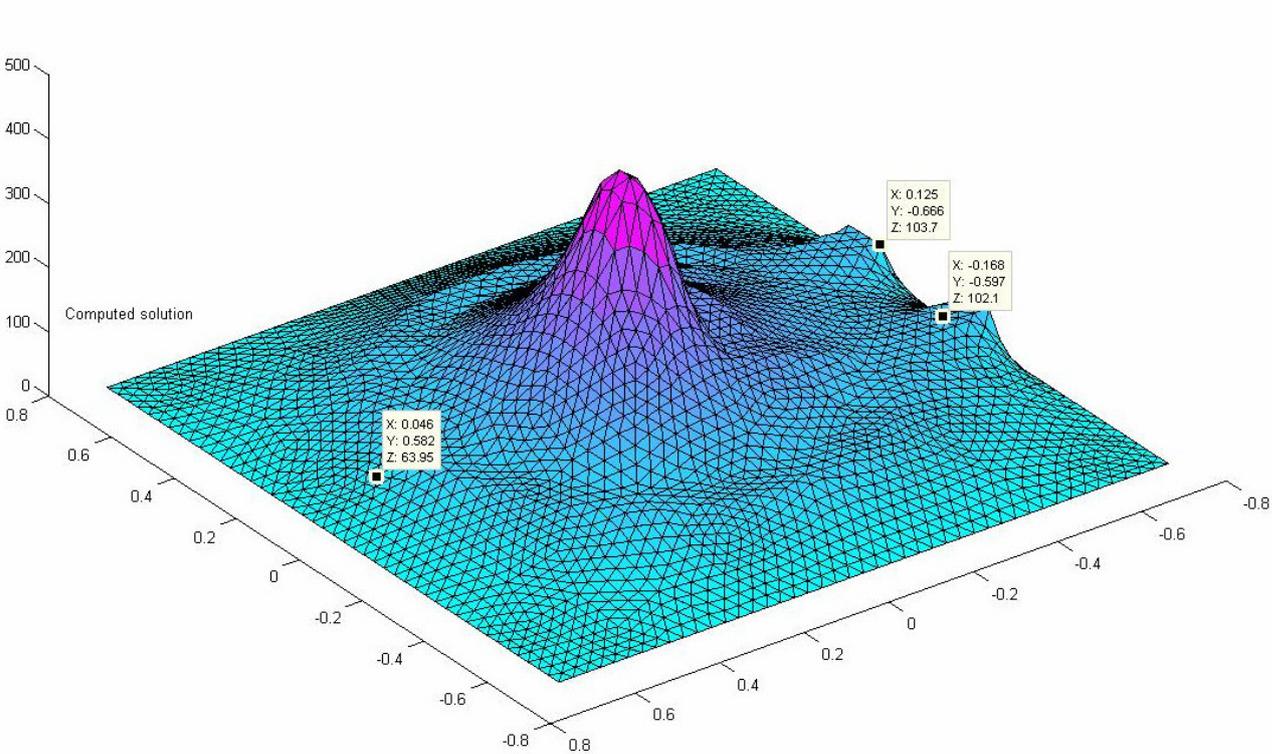


Figure 6: Computed solution for the whole system (AGGA)

As you can see, the temperature of the aluminum is approximately 105 °C to 100 °C, which indicates that the temperature that is gained from the filament is $\cong 103$ °C.

Here is a figure that shows the solution to the whole model (AGGA).

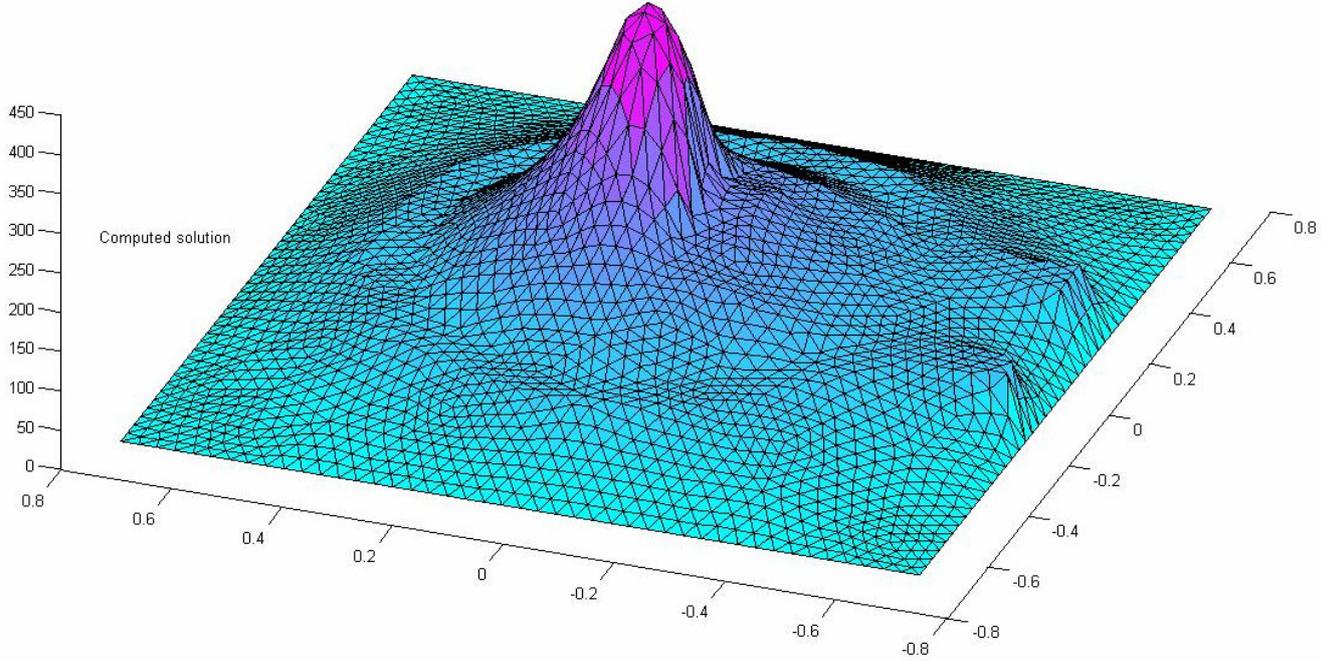


Figure 7: Computed solution for the whole system (AGGA) at the other side

5.1.2. Temperature gained from current flow

When electric current flows through a wire of metal the wire becomes hot, and if the metal is of high resistance then the temperature that is generated will be higher, (the higher the resistance the higher the temperature), and this is known as the Joule Heating Law, which states that the current flowing through wires encounters resistance, and that some electrical energy is transformed into thermal energy through it.. In mathematical form Joule's law is given by:

$$P_{joule} = I^2 R \quad (20)$$

Where I is the current (ampere), R is the material resistance (ohm) and P is the power (watt).

The resistance is given by

$$R = \frac{\rho L}{A}, \quad (21)$$

Where:

ρ : is the resistivity of aluminum.

L : is length of the aluminum piece.

A : is the cross-section area of the aluminum piece.

First we need to calculate the value of resistivity that is changing with temperature, and the formula of resistivity ρ is given by:

$$\rho = \rho_0 (1 + \alpha(T - T_0)), \quad (22)$$

Where ρ_0 is the resistivity of aluminum at room temperature ($20C^\circ$) and it is $3.17 \times 10^{-8} \text{ ohm}$, $\alpha = 0.0039$ is the temperature coefficient of aluminum, T_0 and T are the temperatures of aluminum

before and after heating, so we can consider $T_0 = 20^\circ C$ as the room temperature and $T = 103^\circ C$ as the temperature of aluminum after it has been heated from the filament as it is simulated in the whole system, substituting this in equation (22), we get the resistivity $\rho = 4.2 \times 10^{-8} \text{ ohm}$.

Now, the aluminum piece will be considered as a parallelogram plate with length $L = 10 \times 10^{-2} m$, and cross-section $A = 0.882 \times 10^{-7} m^2$ (thin plates with width $= 2 \times 10^{-2} m$), now we can calculate the resistance of the aluminum plates from equation (21), and it will be $4.762 \times 10^{-2} \text{ ohm}$.

By Joule's law for heating (equation 22) and consider that the value of the electrical current that is usually used in light bulb is 3 ampere at the average; it is easy to calculate the power that is generated in the aluminum plates $P_{joule} = I^2 R = 3^2 \times 4.762 \times 10^{-2} = 0.4286 \text{ watts}$. And if we turn on the bulb for one hour we will get the heat that is generated in the plates,

$$H = P \times t = 0.4286 \text{ watt} \times 1 \text{ hour} = 0.4286 \text{ Joule}.$$

(This is the heat that is generated in the plates).

But the heat that is generated in the material is proportional to the change of temperature and the mass of the body, i.e. $H \propto M \times \Delta T$, where M is the mass of the plates, and ΔT is the temperature change value of the plates, this relation gives:

$$H = c \times M \times \Delta T, \quad (23)$$

Here c is the heat capacity of the aluminum and it is equal to $900 J / kg \cdot ^\circ C$, and the mass can be calculated by the usual mass law, $m = d \times v$ ($d = 2700 kg / m^3$ is the density of aluminum, and v is the volume of the aluminum plates that is easy to calculate, $v = A \times L = 10 \times 10^{-2} m \times 0.882 \times 10^{-7} m^2 = 8.820 \times 10^{-9} m^3$), hence the mass is equal to $2.3814 \times 10^{-5} kg$.

Substitute all results in equation (23), we will get the value of the temperature change affected by the current flow,

$$H = c \times M \times \Delta T \Rightarrow 0.4286 = 900 \times 2.3814 \times 10^{-5} \times \Delta T \Rightarrow \Delta T = 19.9960^\circ C \approx 20^\circ C. \text{ This is the temperature that is generated from the current flow.}$$

Therefore at Γ_1 the temperature will be defined as

$$T_{\Gamma_1} = T_{filament} + T_{current} = 103 + 20 = 123^\circ C.$$

5.2. boundary Γ_2 and Γ_3

At boundaries Γ_2 and Γ_3 , there are some changes of temperature between the glass and the air surrounding it, hence we can consider Neumann conditions at those boundaries, and it can be obtained by the continuity of the solution at the surface of the glass, which implies: $a_2 \nabla T_{glass} = a_1 \nabla T_{air}$, and $a_1 \nabla T_{air}$ is proportional to the difference of temperature between the two media, this means that

$a_1 \nabla T_{air} = k(T_{glass} - T_{air})$, the temperature of the air outside is taken to be the temperature of the room at $20^\circ C$.

5.3. Final model

Find the temperature $u(x, y)$ of the system consisting of the gas and the glass together such that:

$$\begin{cases} -a\Delta u = f & \text{in } \Omega, \\ u = 103 & \text{on } \Gamma_1, \\ -a\partial_n u = u - 20 & \text{on } \Gamma_2, \text{ and } \Gamma_3 \end{cases} \quad (24)$$

Where a is defined as:

$$\begin{cases} 0.0314, & x^2 + y^2 < 0.44^2, \\ 1, & 0.4^2 \leq x^2 + y^2 \leq 0.5^2, \end{cases} \quad (25)$$

And the function source is given by $f(x, y) = 1700$.

6.0. Coordinates of Laplacian equation

The Laplacian operator in spherical coordinates is

$$\nabla^2_{sph} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \quad (26)$$

The derivation is fairly straight forward and begins with locating a vector r in spherical coordinates as shown in the figure.

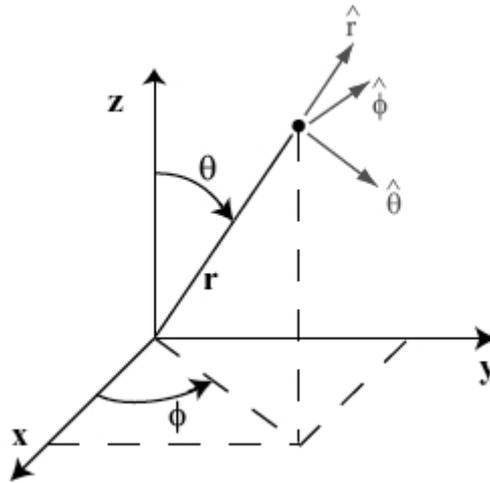


Figure 8: Spherical Coordinates

The z component of the unit vector in direction of r is given from the simple right triangle

$$\cos \theta = \frac{z}{|\hat{r}|}$$

Since a unit vector has a length of 1, the z component is

$$z = \cos \theta$$

To get the x component, we need to get the \hat{r} projected onto the xy - plane

$$\cos(90 - \theta) = \frac{|\hat{r}_{xy}|}{|\hat{r}|}$$

Using the trig identity

$$\cos(90 - \theta) = \sin \theta$$

The projected unit vector is

$$|\hat{r}_{xy}| = \sin \theta$$

Finally, the x component is reached through the right triangle

$$\cos \phi = \frac{x}{|\hat{r}_{xy}|}$$

Giving

$$x = \cos \phi \sin \theta$$

The y component follows the x component through

$$\sin \phi = \frac{y}{|\hat{r}_{xy}|}$$

This yield

$$y = \sin \phi \sin \theta$$

The vector in spherical coordinates is then

$$R = r\hat{r} = r[\sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k}] \quad (27)$$

Now describing the unit vectors of a moving particle $\hat{r}, \hat{\theta}, \hat{\phi}$ shown in the spherical coordinates figure is a little trickier. If a particle moves in the \hat{r} direction, it only moves in or out along r so

$$\frac{\partial R}{\partial r} = k\hat{r}$$

For $\hat{\theta}$ and $\hat{\phi}$, think of uniform circular motion like a record, so they can be calculated from

$$\hat{\theta} = \frac{\partial R / \partial \theta}{|\partial R / \partial \theta|}$$

$$\hat{\phi} = \frac{\partial R / \partial \phi}{|\partial R / \partial \phi|}$$

Next, take the partial derivatives of (27) and then calculate their magnitudes to get the above unit vectors.

$$\frac{\partial R}{\partial \theta} = r[\cos \theta \cos \phi \hat{i} + \cos \theta \sin \phi \hat{j} - \sin \theta \hat{k}]$$

$$\frac{\partial R}{\partial \theta} = r[-\sin \theta \sin \hat{\phi} + \sin \theta \cos \hat{\phi}]$$

$$\left| \frac{\partial R}{\partial \theta} \right| = \sqrt{x \cdot x} = \sqrt{r^2 [\cos^2 \theta \cos^2 \phi + \cos^2 \theta \sin^2 \phi + \sin^2 \theta]}$$

$$\left| \frac{\partial R}{\partial \theta} \right| = r \sqrt{\cos^2 \theta (\cos^2 \phi + \sin^2 \phi) + \sin^2 \theta}$$

Using the basic trig identity

$$\left| \frac{\partial R}{\partial \theta} \right| = r$$

$$\left| \frac{\partial R}{\partial \phi} \right| = r \sqrt{\sin^2 \theta (\sin^2 \phi + \cos^2 \phi)}$$

$$\left| \frac{\partial R}{\partial \phi} \right| = r \sin \theta$$

Then plug in these values to get

$$\hat{\phi} = -\sin \phi \hat{i} + \cos \phi \hat{j}$$

$$\hat{\theta} = \cos \theta \cos \phi \hat{i} + \cos \theta \sin \phi \hat{j} - \sin \theta \hat{k}$$

And from before we had

$$\hat{r} = \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k}$$

The generalized differential for curvilinear coordinates is

$$dR = \frac{\partial R}{\partial u_1} du_1 + \frac{\partial R}{\partial u_2} du_2 + \frac{\partial R}{\partial u_3} du_3$$

For spherical coordinates we have

$$u_1 = r$$

$$u_2 = \theta$$

$$u_3 = \phi$$

And from earlier we learned

$$\left| \frac{\partial R}{\partial r} \right| \hat{r} = \frac{\partial R}{\partial r}$$

$$\left| \frac{\partial R}{\partial r} \right| \hat{\theta} = \frac{\partial R}{\partial \theta}$$

$$\left| \frac{\partial R}{\partial r} \right| \hat{\phi} = \frac{\partial R}{\partial \phi}$$

Plugging these values into the generalized differential yields

$$dR = dr\hat{r} + r d\theta\hat{\theta} + r \sin\theta d\phi\hat{\phi}$$

The next major step is to see how the gradient fits into definition of the differential for a function f

$$df = \frac{\partial f}{\partial r} dr + \frac{\partial f}{\partial \theta} d\theta + \frac{\partial f}{\partial \phi} d\phi$$

$$df = \nabla f \cdot dR$$

So we see that the following must be equal and we need to solve for the gradient's components.

$$\frac{\partial f}{\partial r} dr = \nabla_r f dr$$

$$\frac{\partial f}{\partial \theta} d\theta = \nabla_\theta f d\theta$$

$$\frac{\partial f}{\partial \phi} d\phi = \nabla_\phi f d\phi$$

Therefore our scale factors are

$$\nabla_r = 1$$

$$\nabla_\theta = \frac{1}{r}$$

$$\nabla_\phi = \frac{1}{r \sin\theta}$$

This finally gives us the gradient in spherical coordinates

$$\nabla_{sph} = \frac{\partial}{\partial r} \hat{r} + \frac{\partial}{\partial \theta} \frac{\hat{\theta}}{r} + \frac{\partial}{\partial \phi} \frac{\hat{\phi}}{r \sin\theta}$$

The last tedious calculation is then the Laplacian, which is our goal

$$\Delta = \nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial}{\partial r} \hat{r} + \frac{\partial}{\partial \theta} \frac{\hat{\theta}}{r} + \frac{\partial}{\partial \phi} \frac{\hat{\phi}}{r \sin\theta} \right) \cdot \left(\frac{\partial}{\partial r} \hat{r} + \frac{\partial}{\partial \theta} \frac{\hat{\theta}}{r} + \frac{\partial}{\partial \phi} \frac{\hat{\phi}}{r \sin\theta} \right)$$

Let us break it up by components to make it easy to view, so carrying out only part of the dot product our 1st term is

$$\hat{r} \cdot \left(\frac{\partial \hat{r}}{\partial r} \frac{\partial}{\partial r} + \hat{r} \frac{\partial^2}{\partial r^2} + \frac{\partial \hat{\theta}}{\partial r} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\theta} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\theta}}{r} \frac{\partial^2}{\partial r \partial \theta} + \frac{\partial \hat{\phi}}{\partial r} \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} + \hat{\phi} \frac{\partial}{\partial r} \left(\frac{1}{r \sin\theta} \right) \frac{\partial}{\partial \phi} + \frac{\hat{\phi}}{r \sin\theta} \frac{\partial^2}{\partial r \partial \phi} \right)$$

While this looks a little scary, all but the 2nd term is zero. The first term is zero because there is no r in \hat{r}

$$\hat{r} = \sin\theta \cos\phi \hat{i} + \sin\theta \sin\phi \hat{j} + \cos\theta \hat{k}$$

So by taking the derivative with respect to r this yields zero. Similarly, the 3rd and 6th term are zero when taking the partial derivatives. The 4th, 5th, 7th and 8th terms are zero because the dot product of two orthogonal vectors (90°) is zero so

$$\hat{r} \cdot \hat{\theta} = 0$$

$$\hat{r} \cdot \hat{\phi} = 0$$

This leaves only the second term

$$\hat{r} \cdot \hat{r} \frac{\partial^2}{\partial r^2} = \frac{\partial^2}{\partial r^2} \quad (28)$$

Now onto the $\hat{\theta}$ term

$$\frac{\hat{\theta}}{r} \cdot \left(\frac{\partial \hat{r}}{\partial \theta} \frac{\partial}{\partial r} + \hat{r} \frac{\partial^2}{\partial \theta \partial r} + \frac{\partial \hat{\theta}}{\partial \theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\theta} \frac{\partial}{\partial \theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\theta}}{r} \frac{\partial^2}{\partial \theta^2} + \frac{\partial \hat{\phi}}{\partial \theta} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} + \hat{\phi} \frac{\partial}{\partial \theta} \left(\frac{1}{r \sin \theta} \right) \frac{\partial}{\partial \phi} + \frac{\hat{\phi}}{r \sin \theta} \frac{\partial^2}{\partial \theta \partial \phi} \right)$$

The dot product gets rid of the 2nd, 3rd, 7th and 8th terms, while the 4th and 6th terms are zero when taking the derivatives

$$\frac{\partial}{\partial \theta} \frac{1}{r} = 0$$

$$\frac{\partial \hat{\phi}}{\partial \theta} = 0$$

Two terms remain, for the first term we need to calculate

$$\frac{\partial \hat{r}}{\partial \theta} = \cos \theta \cos \hat{\phi} \hat{i} + \cos \theta \sin \hat{\phi} \hat{j} - \sin \theta \hat{k} = \hat{\theta}$$

This yield

$$\frac{\hat{\theta}}{r} \cdot \hat{\theta} \frac{\partial}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \quad (29)$$

The 5th term is just the dot product

$$\frac{\hat{\theta}}{r} \cdot \frac{\hat{\theta}}{r} \frac{\partial^2}{\partial \theta^2} = \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \quad (30)$$

Finally, the $\hat{\phi}$ part of the dot product is

$$\frac{\hat{\phi}}{r \sin \theta} \cdot \left(\frac{\partial \hat{r}}{\partial \phi} \frac{\partial}{\partial r} + \hat{r} \frac{\partial^2}{\partial \phi \partial r} + \frac{\partial \hat{\theta}}{\partial \phi} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\theta} \frac{\partial}{\partial \phi} \frac{1}{r} \frac{\partial}{\partial \theta} + \frac{\hat{\theta}}{r} \frac{\partial^2}{\partial \phi \partial \theta} + \frac{\partial \hat{\phi}}{\partial \phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} + \hat{\phi} \frac{\partial}{\partial \theta} \left(\frac{1}{r \sin \theta} \right) \frac{\partial}{\partial \phi} + \frac{\hat{\phi}}{r \sin \theta} \frac{\partial^2}{\partial \phi^2} \right)$$

Once again the dot product makes the 2nd, 4th and 5th terms zero. The first term derivative

$$\frac{\partial \hat{r}}{\partial \phi} = -\sin \theta \sin \hat{\phi} \hat{i} + \sin \theta \cos \hat{\phi} \hat{j} = \sin \theta \hat{\phi} \frac{\partial}{\partial r}$$

This leads to

$$\frac{\hat{\phi}}{r \sin \theta} \cdot \sin \theta \hat{\phi} \frac{\partial}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \quad (31)$$

The 3rd term derivative is

$$\frac{\partial \hat{\theta}}{\partial \phi} = -\cos \theta \sin \hat{\phi} \hat{i} + \cos \theta \cos \hat{\phi} \hat{j} = \cos \theta \hat{\phi}$$

This leads to

$$\frac{\hat{\phi}}{r \sin \theta} \bullet \cos \theta \hat{\phi} \frac{1}{r} \frac{\partial}{\partial \theta} = \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \quad (32)$$

The 6th term can be seen to be zero because the derivative of $\hat{\phi}$ with respect to ϕ is a vector perpendicular to $\hat{\phi}$ (feel free to carry out this calculation), so the dot product will be zero.

The 7th term derivative is zero

$$\frac{\partial}{\partial \phi} \left(\frac{1}{r \sin \theta} \right) = 0$$

Then we keep the 8th term

$$\frac{\hat{\phi}}{r \sin \theta} \bullet \frac{\hat{\phi}}{r \sin \theta} \frac{\partial^2}{\partial \phi^2} = \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \quad (33)$$

Putting the results from (28),(29),(30),(31),(32) and (33) we get

$$\nabla^2_{sph} = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

Certainly, we could finish here, but let us combine some terms and notice the following relationships (check these for yourself)

$$\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$$

$$\frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial}{\partial \theta} = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)$$

This gives us the equation given in (26), the Laplacian in spherical coordinates

$$\nabla^2_{sph} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$$

7.0. Finite Element Method (FEM)

FEM was developed initially, and prospered, as a computer based simulation method for the analysis of aerospace structure. Then it found its way into both design and analysis of complex structural systems, not only in aerospace but in civil and mechanical engineering. In the late 1960s it expanded to the simulation of non-structural problems in fluids, thermo mechanics and electromagnetics. This “physical FEM” is an operational tool, which fits primarily the operational knowledge component of engineering, and draws from the mathematical models of the real world.

The success of FEM as a general-purpose simulation method attracted attention in the 1970s from two quarters beyond engineering: mathematics and software entrepreneurs. The world of

FEM eventually split into applications, mathematics and commercial software products. The former two are largely housed in the comfortable obscurity of academia.

7.1. Theory

FEM is one of the methods that give the numerical solution to partial differential equations (PDE), and it depends on the triangulation τ_h of the space Ω , where $\tau_h = \{K, \text{ such that } \cup K = \Omega\}$,

The final model of the problem is to simulate the solution $u(x, y)$ such that:

$$\begin{cases} -a\Delta u = f & \text{in } \Omega, \\ u = 103 & \text{on } \Gamma_1, \\ -a\partial_n u = u - 20 & \text{on } \Gamma_2, \Gamma_3 \end{cases}$$

The variation formulation is based on multiplying the equation $-a\Delta u = f$ by test function $v \in V$, where $V = \{v : \int ((v')^2 + (v)^2) dx < \infty, \text{ and } v = 0, \text{ in } \Gamma_1\}$, and integrate over the whole Ω , we get

$$\begin{aligned} \Rightarrow \int_{\Omega} (-a\Delta u)v &= -\int_{\Omega} a(\nabla \cdot \nabla u)v \, dx \\ &= -\int_{\Omega} a(\nabla \cdot v \nabla u) \, dx + \int_{\Omega} a \nabla u \cdot \nabla v \, dx \quad (\text{by Green's Theorem}) \\ &= -\int_{\partial\Omega} a v \nabla u \cdot \vec{n} \, ds + \int_{\Omega} \nabla u \cdot \nabla v \, dx \\ &= -\int_{\Gamma_1} a \frac{\partial u}{\partial n} v \, ds - \int_{\Gamma_2} a \frac{\partial u}{\partial n} v \, ds + \int_{\Omega} a \nabla u \cdot \nabla v \, dx \quad (\text{choose } v = 0 \text{ on } \Gamma_1) \\ &= \int_{\Omega} a \nabla u \cdot \nabla v \, dx + \int_{\Gamma_2} (20 - u)v \, ds \\ &= \int_{\Omega} f v \, dx \end{aligned}$$

Finite Element Method is read as find $U \in V_h$ such that for every $v \in V_h$ the following hold

$$\begin{aligned} \int_{\Omega} (-a\Delta U)v &= -\int_{\Omega} a(\nabla \cdot \nabla U)v \, dx \\ \Rightarrow \int_{\Omega} a \nabla U \cdot \nabla v \, dx &+ \int_{\Gamma_2} (20 - U)v \, ds = \int_{\Omega} f v \, dx \end{aligned}$$

Where $V_h = \{v : v \text{ is a piecewise continuous linear function in each } \tau_h, \text{ and } v = 0 \text{ on } \Gamma_1\}$

7.2. Stability estimate

To derive stability estimate for the model, and to make this reasonable it is better to take the bulb and the air surrounding it as a complete model, so the temperature of the air will be considered as a boundary condition of the system. This means that it will be Dirichlet conditions at the boundaries,

hence the equation of the system is given by:

$$\begin{cases} -a\Delta u = f, & \text{in } \Omega, \\ u = 20, & \text{on } \Gamma, \end{cases}$$

Multiply the equation $-a\Delta u = f$ by a test function $v = u$, (take $a = 1$ for simplicity) and integrate over the Ω we get

$$\begin{aligned} \Rightarrow \int_{\Omega} (-\Delta u)u &= -\int_{\Omega} (\nabla \cdot \nabla u)u \, dx \\ &= -\int_{\Omega} (\nabla \cdot u \nabla u) \, dx + \int_{\Omega} \nabla u \cdot \nabla u \, dx \quad (\text{by Green's Theorem}) \\ &= -\int_{\partial\Omega} a u \nabla u \cdot \vec{n} \, ds + \int_{\Omega} \nabla u \cdot \nabla u \, dx \\ &= -\int_{\Gamma} \frac{\partial u}{\partial n} u \, ds + \int_{\Omega} \nabla u \cdot \nabla u \, dx \quad (\text{choose } u = 0 \text{ on } \Gamma) \\ &= \int_{\Omega} \nabla u \cdot \nabla u \, dx \\ &= \int_{\Omega} f \cdot u \, dx \end{aligned}$$

$$\Rightarrow \int_{\Omega} (\nabla u)^2 \, dx = \int_{\Omega} f \cdot u \, dx, \quad x \in \Omega, \text{ and } u \in V$$

$$\Rightarrow \|\nabla u\|^2 = \int_{\Omega} f \cdot u \, dx \leq \|f\| \|u\|, \text{ Where } \|\cdot\| \text{ denotes the usual } L_{2(\Omega)} \text{ - norm}$$

And by Poincaré inequality $\|u\| \leq C_{\Omega} \|\nabla u\|$ for some constant C_{Ω} depending on the boundary of the space, which yields:

$$\|\nabla u\| \leq C_{\Omega} \|f\| \Rightarrow \|u\| \leq C_{\Omega} \|\nabla u\| \leq C_{\Omega}^2 \|f\|$$

This means that the solution $u(x, y)$ is less than the given source.

8.0. Life-time

When an incandescent lamp is working, it produces heat and light because electrons are running through a thin filament and thereby rising its temperature. The amount of light and heat emitted will increase with temperature.

Lamps are designed to work at a specific voltage (U). By choosing the right values of filament length (l) and cross-sectional area (A) the resistance (R) is determined. Voltage and resistance gives the current (I) and the power (P) output.

$$R = \rho_w \frac{l}{A}$$

$$I = \frac{U}{R} \quad (\text{Ohm's first law})$$

$$P = UI = \frac{U^2}{R} = RI^2$$

$$\rho_w = \rho_0 (1 + \alpha(T_w - T_0))$$

$\rho \sim \text{resistivity}$

$\alpha \sim \text{temperature coefficient}$

The power fraction that produces light increases with temperature and that is why tungsten is chosen for filament material. Tungsten has an abnormally high melting temperature. Even so, at the operational temperature of a lamp tungsten may burn in the presence of oxygen. Because of that the bulb is evacuated. At these high temperatures there will also be considerable evaporation of tungsten and the lamp will go dimmer. The decrease of the radius will increase the resistance and reduce the light output, while the evaporated tungsten deposits on the inside of the glass. Today, most light bulbs have a filling gas to reduce both evaporation and its consequences.

Evaporation depends on temperature and pressure. Inert filling gases such as nitrogen, argon and xenon, procure pressure and will make some of the evaporated tungsten bounce back and redeposit on the filament. Halogen filling gases, on the other hand, will react with the evaporated tungsten, both at the glass surface and in the bulb space, and when the compound reaches the hottest parts of the filament break and redeposit the tungsten.

Evaporation is not uniform along the filament, because the temperature is not constant along its length. The hinges and support wires will remove some heat by way of conduction and the radius of the filament may not be altogether uniform. Thinner parts of the filament will have a greater resistance than the average, since the same current flows through all the length of the filament. This will raise the temperature because the moving electrons will collide more frequently and transfer part of their kinetic energy to vibrational energy in the tungsten atoms, i.e. heat. Additionally, the smaller cross-sectional area will be able to radiate less heat. Accordingly, these thin 'hot-spots' will evaporate increasingly more tungsten until one of them breaks/melts.

The reason this often happens as the lamp is turned on is because tungsten has less resistance when cool than when heated. The thin spots will reach working temperature faster, because of the smaller mass, and the high current resulting from the low resistance in the greater part of the filament will make the hot-spots even hotter and the evaporation greater, compared to during steady-state operation.

The rate of mass evaporation per unit area (J) can be expressed as:

$$J = \frac{1}{A} \frac{dm}{dt} = \frac{1}{\pi 2rl} \frac{d(\pi r^2 l D)}{dt} = \frac{dr}{dt} D$$

$m \sim \text{mass}$

$D \sim \text{density}$

The temperature dependence of the mass evaporation rate can be found labelled (eg. in the *CRC Handbook of Physics and Chemistry* E-364, (1978)).

The rate of redeposition of tungsten in a bulb with an inert gas will be proportionate to the probability of colliding with the gas atom (noble gas) or molecule (nitrogen). Assuming the gas in the bulb at room temperature (T) has the pressure (p) of 1 atm the amount (n) of gas atoms/molecules in the bulb can be obtained.

$$\frac{p}{R_g T} = \frac{n}{V} \quad \text{Ideal gas law}$$

$R_g \sim \text{molar gas const.}$

The atomic radius of the gas atom or the covalent radius of the gas molecule can be used together with the amount to calculate the total volume of the gas particles (V_p). Taking the bulb to be a sphere of radius r_b and volume V_b , the filament to be centrally placed and assuming a collision with a gas particle always brings the tungsten back to the filament at the spot of evaporation, the change of radius can be expressed as:

$$\frac{dr}{dt} = -\frac{J}{D} \left(\frac{\left(1 - \frac{V_p}{V_b} \right)^{\frac{r_b}{2r_w}}}{V_w} \right)$$

The expression in the bracket is the probability for a tungsten atom to propagate a step of length equal to its own diameter towards the glass without colliding. This expression is however clearly wrong in more than one aspect, and we decided to model the lifetime in a vacuum bulb.

Breaking of the filament is assumed to take place when the thinnest spot has reached the melting temperature of tungsten or is sufficiently thin to break due to mechanical causes.

9.0. Results and discussion

Now we show some of the results that are obtained after solving the model numerically by using a Matlab code, the solution $u(x, y)$ represents the temperature of the glass and the gas inside.

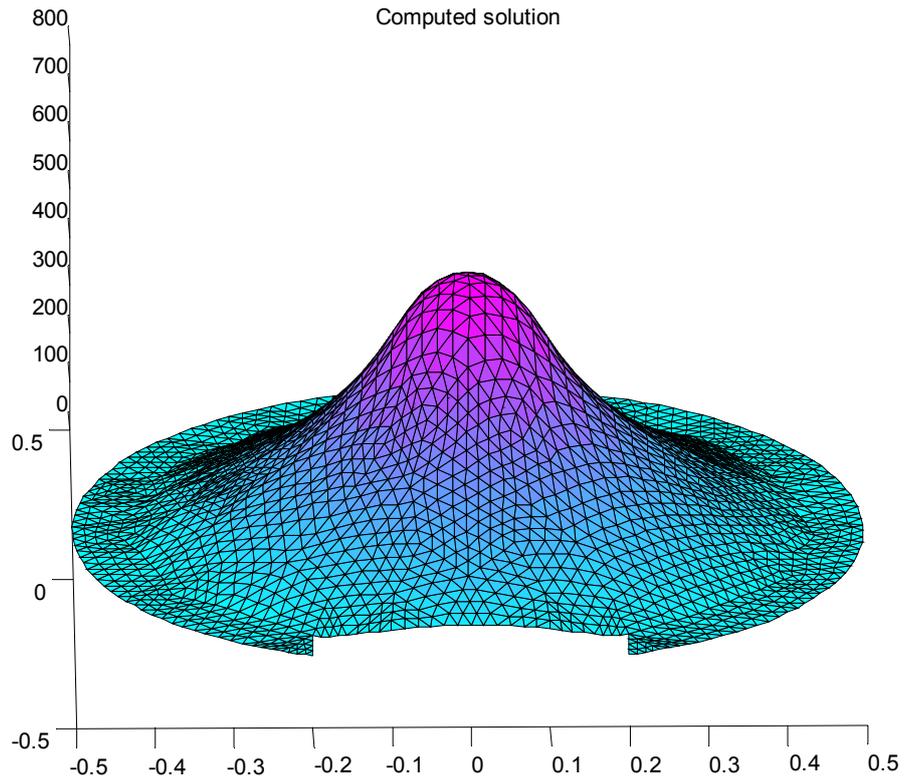


Figure 9: Computed solution $U(x,y)$ for the glass and the gas

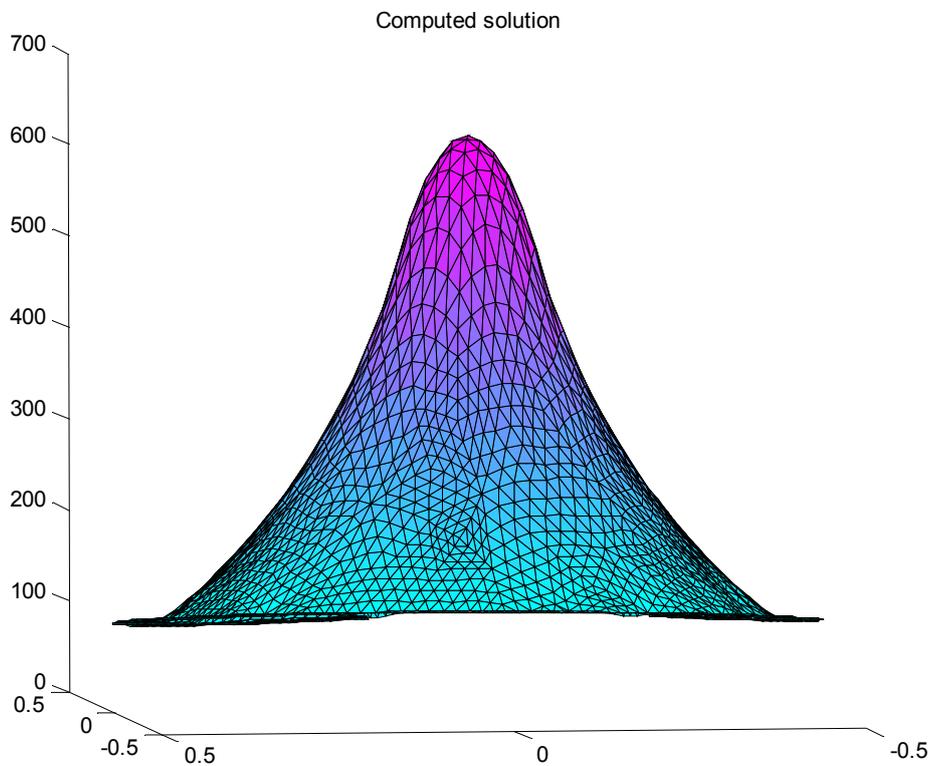


Figure 10: Computed solution $U(x,y)$ for the glass and the gas

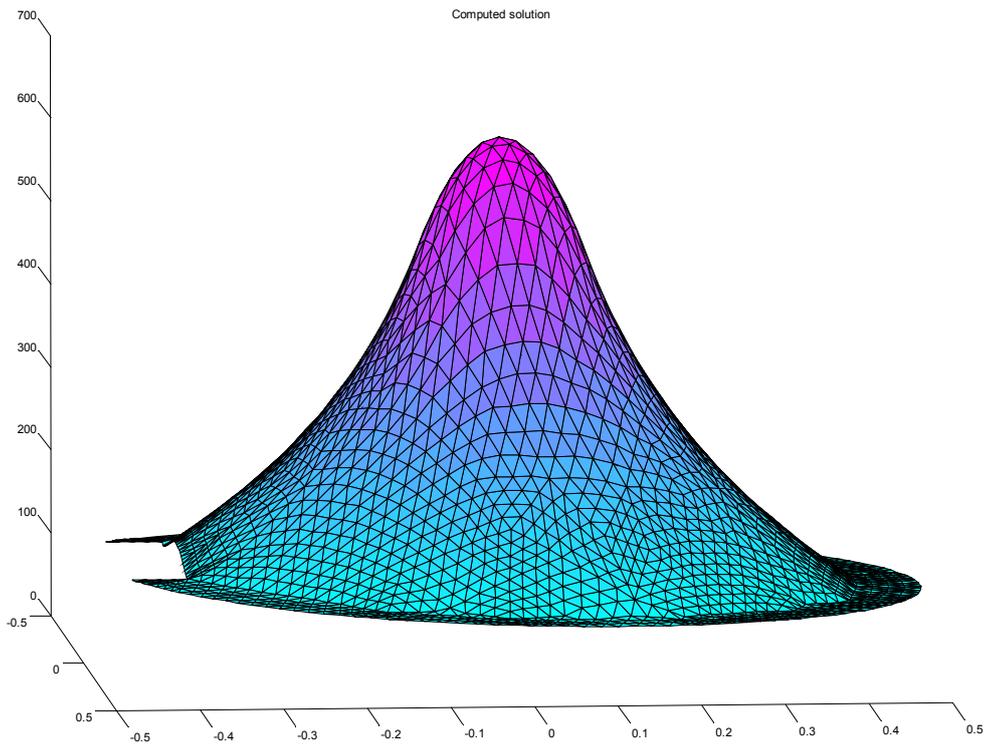


Figure 11: Computed solution $U(x,y)$ for the glass and the gas

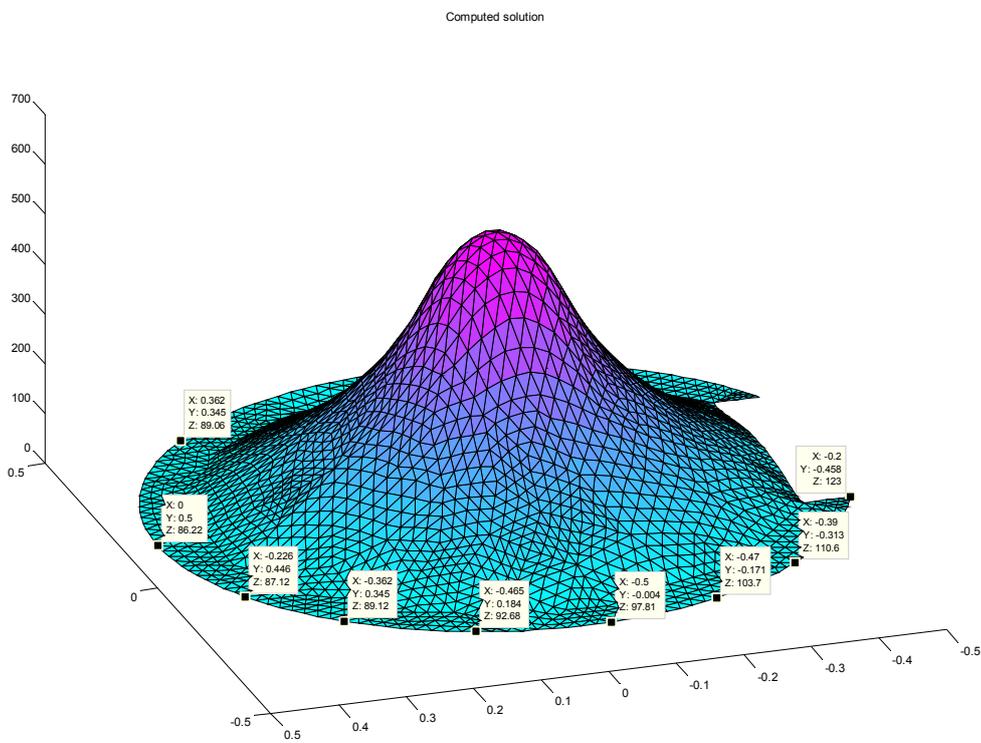


Figure 12: Computed solution $U(x,y)$ for the glass and the gas

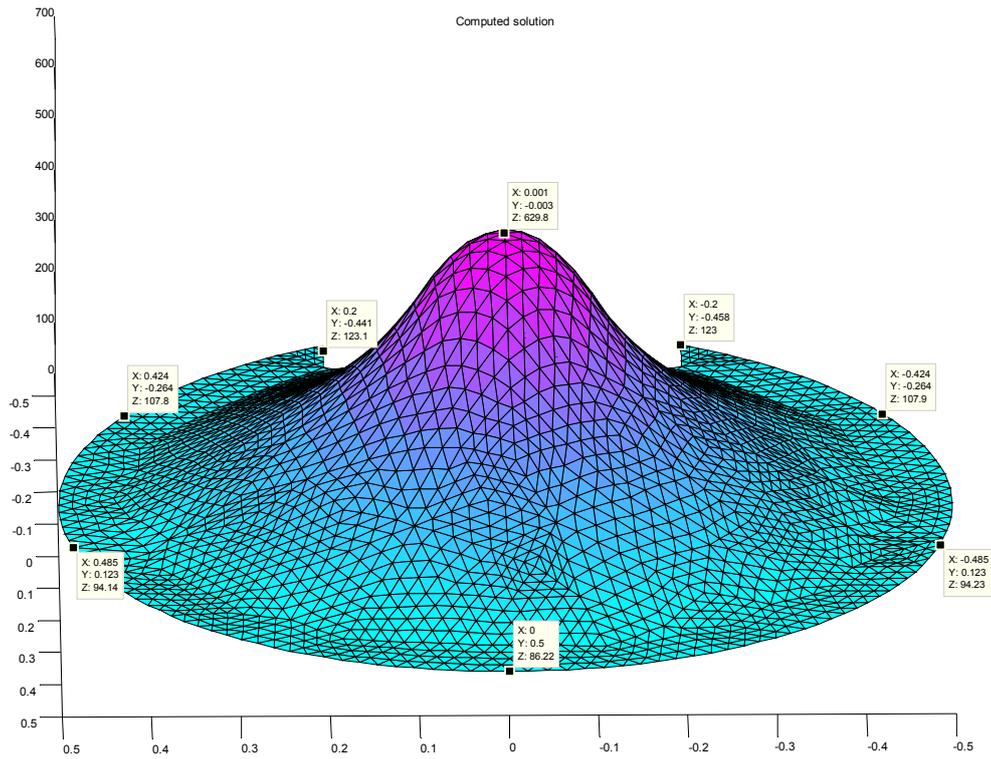


Figure 13: Computed solution $U(x,y)$ for the glass and the gas

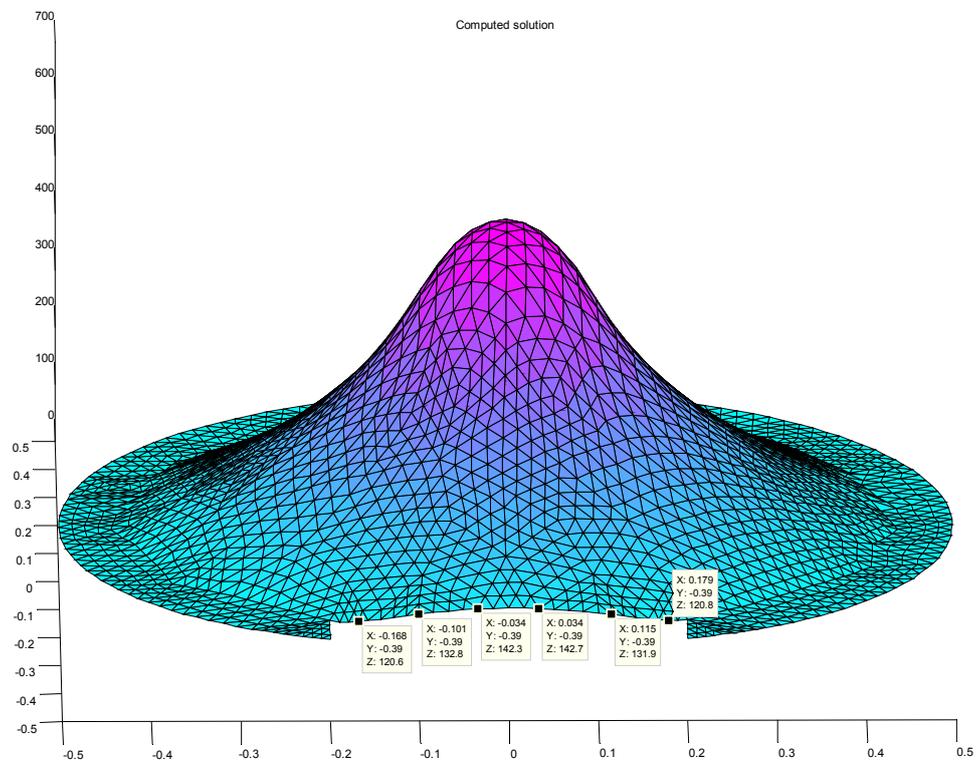


Figure 14: Computed solution $U(x,y)$ for the glass and the gas

It is noticed from the figures above (figures: 9-14) that the temperature of the system is high for the gas and a bit lower for the glass, and this is due to the position and the thermal conductivity coefficient of the material, since the temperature of the body is proportional to its heat conductivity, and also the temperature has an opposite proportionality to the distance.

Aluminum is of high thermal conductivity, so it conducts temperature perfectly to the contact material, and it is obvious from the figures above that the part of the glass near the aluminum is more heated than the farther part, and the glass temperature is continuously decreased when we depart away from the aluminum.

The temperature of the glass is approximately arranged between $123^{\circ}C$ (contact points to the aluminum) to $86.22^{\circ}C$ (farthest point from the aluminum), you can see also the symmetry of the solution at $y - axis$.

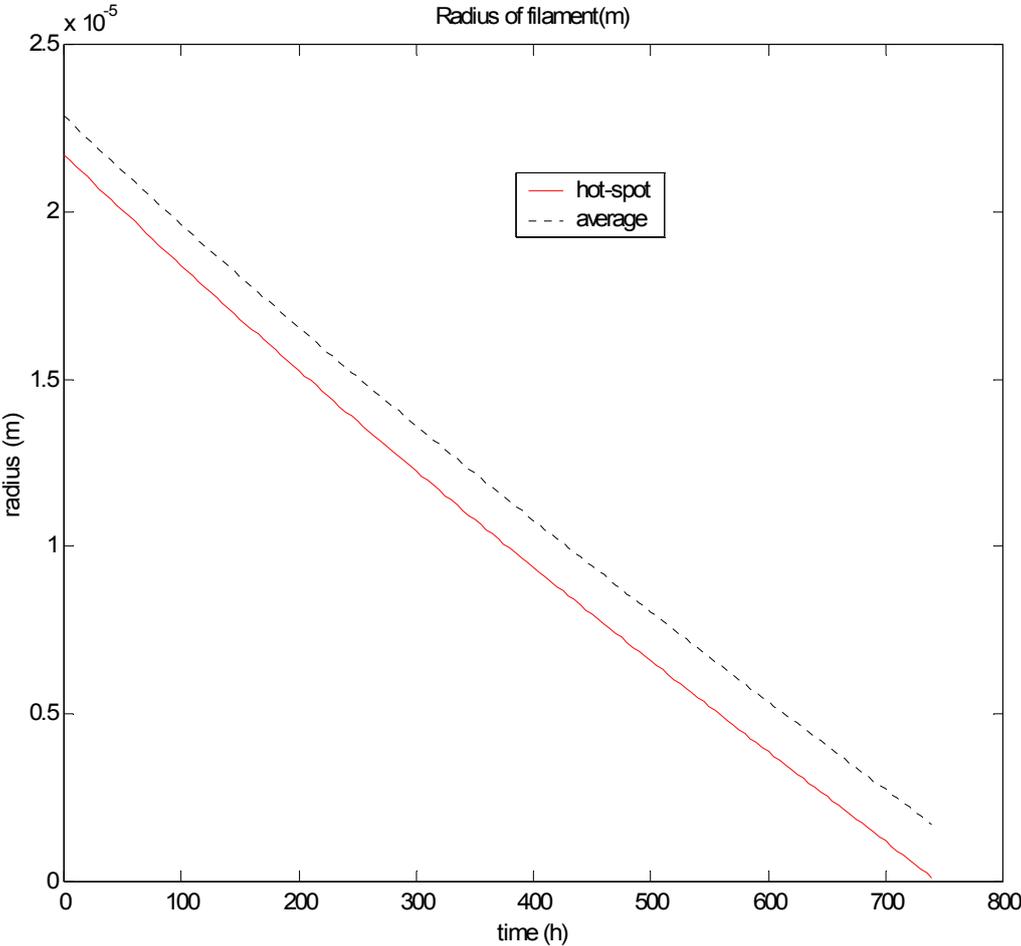


Figure 15: Decrease of the radius of the filament with time.

We chose to model the lifetime of a 60 W bulb designed to work at a voltage of 120 V. The tungsten filament of the modeled bulb has a radius of $22.85 \mu m$ and a length of 0.533 m. We assumed that the

filament would break at a radius of 1 nm in a vacuum bulb and supposed the filament to be manufactured with a tolerated difference in radius of $\pm 5\%$. Accordingly, the critical radius would be 0.95 of the rated radius, i.e. ca 21.71 μm . When modeled like this, the lifetime of the light bulb was found to be ca 740 hours. The rated lifetime of a non-vacuum 60 W bulb with these dimensions of filament is 750 h.

Figure 15 show the change of radius at the hot-spot with time, compared with the change of average radius. It can clearly be seen that the difference between the hot-spot radius and the average radius grows with time. This is consistent with the difference in temperature seen between the hot-spot and the rest of the filament and that the evaporation rate increases when the temperature is increased. When the average cross-sectional area of the filament decreases, the resistance will grow and the current decrease. Since the temperature is dependant on both current a cross-sectional area it will decrease with a flattened curve until some critical point where the cross-sectional area is sufficiently small at the hot-spot to get really hot. When this occurs, the filament is speedily wasting at the hot-spot.

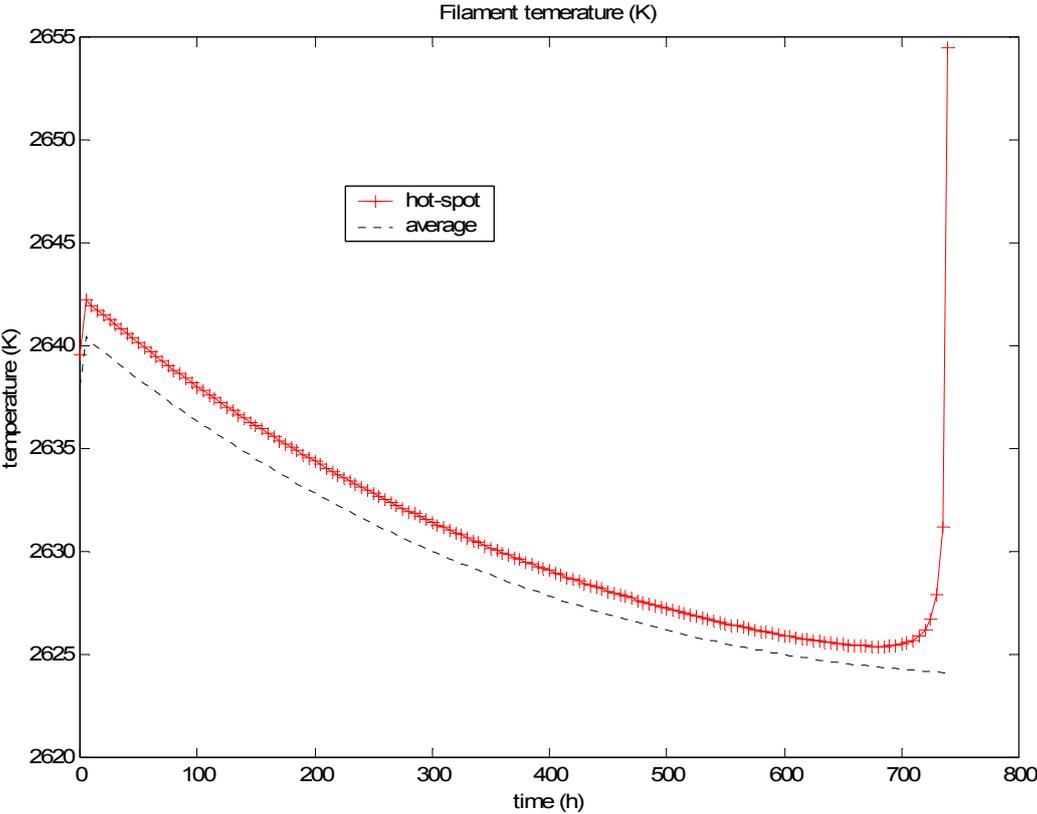


Figure 16: Change in filament temperature with time.

10.0. Conclusion

The incandescent bulb and the fluorescent bulb are the most widely used kinds of electric bulbs in the

industry because of the importance to every day life.

The formulated mathematical model for the system is basically detailed conduction heat transfer. The light bulb was modeled and the Dirichlet boundary conditions Γ_2 and Γ_3 were taken into account. The temperature gained from the filament and current flows were analyzed. The study also gives a description of the coordinates of the Laplacian equation. The description of Finite Element Method (FEM) and stability estimates were used in the implementation of the problem.

In conclusion, the following observations were made:

A) The heat distribution in the glass is symmetric both at the z-axis and the y-axis, and can be modeled in one dimension as a second order boundary value problem with aluminum temperature as a boundary conditions.

B) The lifetime of a light bulb depends on the operating temperature, the uniformity, thickness and length of the filament and if/what kind of filling gas is used.

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12.0. Appendices

12.1. AssembleMatrix

```

function A = AssembleMatrix(points, edges, triangles, pde, W, time)
% Assemble a matrix on a given mesh (points, edges, triangles)
% Create matrix
A = zeros(size(points,2));
% Quadrature points
midpoints = [0.5 0.0; 0.5 0.5; 0.0 0.5]';
gausspoints = [(1-1/sqrt(3))/2 (1-1/sqrt(3))/2];
% Iterate over all elements (interior domain)
for triangle = triangles
% Get nodes, coordinates, and domain number
nodes = triangle(1:3);
coord = points(:,nodes);
d = triangle(4);
% Compute Jacobian of map and area of element
J = coord(:,1)*dphi(1)' + coord(:,2)*dphi(2)' + coord(:,3)*dphi(3)';
dx = 0.5 * abs(det(J));
% Assemble matrix
for p = midpoints
x = coord(:,1)*phi(1,p) + coord(:,2)*phi(2,p) + coord(:,3)*phi(3,p);
if ~isempty(W)
w = evalfunctions(W(nodes,:), p);
dw = evalderivatives(W(nodes,:), p);
else
w = [];
dw = [];
end
for i = 1:3
v = phi(i,p);
dv = J' \ dphi(i);
for j = 1:3
u = phi(j,p);
du = J' \ dphi(j);
integral = feval(pde, u, v, w, du, dv, dw, dx, 0, x, d, time, 1) / 3.0;
A(nodes(i),nodes(j)) = A(nodes(i),nodes(j)) + integral;
end
end
end
end
% Iterate over all edges (boundary)
for edge = edges
% Get nodes, coordinates, and domain number
nodes = edge(1:2);
coord = points(:,nodes);
d = edge(5);
% Compute length of edge
ds = norm(coord(:,1) - coord(:,2));
% Assemble matrix
for p = gausspoints
x = coord(:,1)*phiedge(1,p) + coord(:,2)*phiedge(2,p);
if ~isempty(W)
w = evalfunctionsege(W(nodes,:), p);
dw = evalderivativesedge(W(nodes,:), p);
else
w = [];
dw = [];
end
for i = 1:2
v = phiedge(i,p);
for j = 1:2

```

```

u = phiedge(j,p);
integral = feval(pde, u, v, w, [0;0], [0;0], dw, 0, ds, x, d, time, 1) / 2.0;
A(nodes(i),nodes(j)) = A(nodes(i),nodes(j)) + integral;
end
end
end
end
%--- Basis functions on the reference element ---
function y = phi(index, x)
switch index
case 1
y = 1 - x(1) - x(2);
case 2
y = x(1);
case 3
y = x(2);
end
%--- Basis functions on the reference edge ---
function y = phiedge(index, x)
switch index
case 1
y = 1 - x;
case 2
y = x;
end
%--- Gradients of basis functions on the reference element ---
function y = dphi(index)
switch index
case 1
y = [-1; -1];
case 2
y = [1; 0];
case 3
y = [0; 1];
end
%--- Gradients of basis functions on the reference edge ---
function y = dphiedge(index)
switch index
case 1
y = -1;
case 2
y = 1;
end
%--- Evaluate functions at the given quadrature point ---
function w = evalfunctions(W, p)
n = size(W,2);
w = zeros(1,n);
for i = 1:n
w(i) = W(1,i)*phi(1,p) + W(2,i)*phi(2,p) + W(3,i)*phi(3,p);
end
%--- Evaluate derivatives of functions at the given quadrature point ---
function dw = evalderivatives(W, p)
n = size(W,2);
dw = zeros(2,n);
for i = 1:n
dw(:,i) = W(1,i)*dphi(1) + W(2,i)*dphi(2) + W(3,i)*dphi(3);
end
%--- Evaluate functions at the given quadrature point ---
function w = evalfunctionsege(W, p)
n = size(W,2);

```

```

w = zeros(1,n);
for i = 1:n
    w(i) = W(1,i)*phiedge(1,p) + W(2,i)*phiedge(2,p);
end
%--- Evaluate derivatives of functions at the given quadrature point ---
function dw = evalderivativesedge(W, p)
    n = size(W,2);
    dw = zeros(2,n);
    for i = 1:n
        dw(1,i) = W(1,i)*dphiedge(1) + W(2,i)*dphiedge(2);
    end

```

12.2. AssembleVector

```

function b = AssembleVector(points, edges, triangles, pde, W, time)

% Assemble a vector on a given mesh (points, edges, triangles)
% Create matrix
b = zeros(size(points,2),1);
% Quadrature points
midpoints = [0.5 0.0; 0.5 0.5; 0.0 0.5];
gausspoints = [(1-1/sqrt(3))/2 (1-1/sqrt(3))/2];
% Iterate over all elements (interior domain)
for triangle = triangles
    % Get nodes, coordinates, and domain number
    nodes = triangle(1:3);
    coord = points(:,nodes);
    d = triangle(4);
    % Compute Jacobian of map and area of element
    J = coord(:,1)*dphi(1)' + coord(:,2)*dphi(2)' + coord(:,3)*dphi(3)';
    dx = 0.5 * abs(det(J));
    % Assemble matrix
    for p = midpoints
        x = coord(:,1)*phi(1,p) + coord(:,2)*phi(2,p) + coord(:,3)*phi(3,p);
        if ~isempty(W)
            w = evalfunctions(W(nodes,:), p);
            dw = evalderivatives(W(nodes,:), p);
        else
            w = [];
            dw = [];
        end
        for i = 1:3
            v = phi(i,p);
            dv = J' \ dphi(i);
            integral = feval(pde, 0, v, w, [0;0], dv, dw, dx, 0, x, d, time, 2) / 3.0;
            b(nodes(i)) = b(nodes(i)) + integral;
        end
    end
end
% Iterate over all edges (boundary)
for edge = edges
    % Get nodes, coordinates, and domain number
    nodes = edge(1:2);
    coord = points(:,nodes);
    d = edge(5);

    % Compute length of edge
    ds = norm(coord(:,1) - coord(:,2));
    % Assemble matrix
    for p = gausspoints

```

```

x = coord(:,1)*phiedge(1,p) + coord(:,2)*phiedge(2,p);
if ~isempty(W)
    w = evalfunctionsedge(W(nodes,:), p);
    dw = evalderivativesedge(W(nodes,:), p);
else
    w = [];
    dw = [];
end
for i = 1:2
v = phiedge(i,p);
    integral = feval(pde, 0, v, w, [0;0], [0;0], dw, 0, ds, x, d, time, 2) / 2.0;
    b(nodes(i)) = b(nodes(i)) + integral;
end
end
end
%--- Basis functions on the reference element ---
function y = phi(index, x)
switch index
case 1
    y = 1 - x(1) - x(2);
case 2
    y = x(1);
case 3
    y = x(2);
end
%--- Basis functions on the reference edge ---
function y = phiedge(index, x)
switch index
case 1
    y = 1 - x;
case 2
    y = x;
end
%--- Gradients of basis functions on the reference element ---
function y = dphi(index)
switch index
case 1
    y = [-1; -1];
case 2
    y = [1; 0];
case 3
    y = [0; 1];
end
%--- Gradients of basis functions on the reference edge ---
function y = dphiedge(index)
switch index
case 1
    y = -1;
case 2
    y = 1;
end
%--- Evaluate functions at the given quadrature point ---
function w = evalfunctions(W, p)
n = size(W,2);
w = zeros(1,n);

for i = 1:n
w(i) = W(1,i)*phi(1,p) + W(2,i)*phi(2,p) + W(3,i)*phi(3,p);
end
%--- Evaluate derivatives of functions at the given quadrature point ---

```

```

function dw = evalderivatives(W, p)
n = size(W,2);
dw = zeros(2,n);
for i = 1:n
    dw(:,i) = W(1,i)*dphi(1) + W(2,i)*dphi(2) + W(3,i)*dphi(3);
end
%--- Evaluate functions at the given quadrature point ---
function w = evalfunctionsege(W, p)
n = size(W,2);
w = zeros(1,n);
for i = 1:n
    w(i) = W(1,i)*phiedge(1,p) + W(2,i)*phiedge(2,p);
end
%--- Evaluate derivatives of functions at the given quadrature point ---
function dw = evalderivativesedge(W, p)
n = size(W,2);
dw = zeros(2,n);
for i = 1:n
    dw(1,i) = W(1,i)*dphiedge(1) + W(2,i)*dphiedge(2);
end

```

12.3. GasGlass

```

function integral = GasGlass(u, v, w, du, dv, dw, dx, ds, x, d, t, eq)
if eq == 1
    integral = a(x,d,t)*du*dv*dx + g(x,d,t)*u*v*ds;
else
    integral = f(x,d,t)*v*dx + (g(x,d,t)*gd(x,d,t) - gn(x,d,t))*v*ds;
end
%--- Conductivity (penalty factor) ---
function y = g(x, d, t)
if d==1
    y=0;
elseif d==4
    y = 0;
elseif d==5
    y = 0;
elseif d==6
    y = 0;
elseif d==7
    y = 0;
else
    y = 1e7;
end
%--- conductivity coefficient----
function y = a(x, d, t)
if norm(x)< 0.44
    y = 0.0314;
else
    y = 1;
end
%--- Dirichlet boundary condition ----
function y = gd(x, d, t)
%if d == 1
% y =150;
if d==2
    y = 123;
elseif d==3
    y = 123;
else

```

```

y=0;
end
%--- Neumann boundary condition ---
function y = gn(x, d, t)
if d==1
    y=20;
else
    y =20;
end
%--- Right-hand side, source term ---
function y = f(x, d, t)
if norm(x)< 0.1
    y = 1700;
else
    y = 0;
end

```

12.4. Solver

```

% Load the mesh
unibulb
% Assemble matrix
A = AssembleMatrix(p, e, t, 'GasGlass', [], 0);
% Assemble vector
b = AssembleVector(p, e, t, 'GasGlass', [], 0);
% Solve the linear system
U = A \ b;
% Plot solution
figure(1); clf
pdesurf(p,t,U)
shading faceted
title('Computed solution')

```

12.5. Life-time

```

% declaration of variables
clear all;
r = 22.85e-6;           % initial radius of filament (m)
rc = 0.95*r;           % initial radius at critical spot of filament (m)
t_step =5*3600;        % timestep (s)
T0 = 293;               % reference temperature (K)
Tm = 273+3410;         % melting point of tungsten (K)
U = 120;                % Voltage (V)
P = 60;                 % Power rating of light bulb (W)
l = 0.533;              % length of filamnet (m), assumed constant
rho = 5.5e-8;           % resistivity of tungsten at T0 (ohm m)
alfa = 4.9e-3;         % temperature koefficient (1/K)
dw = 19400;             % density of tungsten at T0 (kg/m^3), assumed constant
%-----
rb = 0.035;             % radius of light bulb (m)
V = 4*pi*rb^3/3;       % volume of light bulb (m^3)
rn = 0.074e-9;         % covalent radius of N atom (m)
n = 101325*V/(8.314*T0); % amount of N2-molecules at T0, assuming
                        % normal pressure (mole)
dg = 2*n*6.022e23*rn^3/rb^3; % volume ratio occupied by N2-molecules
rw = 0.178e-12;        % atomic radius of tungsten (m)
Vw = 4*pi*rw^3/3;      % volume of tungsten atom (m^3)

```

```

%-----
% temperatures and evaporation rates from Phys. Educ. 33(1)January 1998:55-
58 (actually from CRC Handbook...)
K =
[10;11;12;13;14;15;16;17;18;19;20;21;22;23;24;25;26;27;28;29;30;31;32;33;34
;35;36;36.55]*100;
E = [5.32e-33 2.17e-29 3.21e-26 4.35e-23 2.51e-21 2.37e-19 1.25e-17 4.17e-
16 8.81e-15 1.41e-13 1.76e-12 1.66e-11 1.25e-10 8.00e-10 ...
4.26e-9 2.03e-8 8.41e-8 3.19e-7 1.10e-6 3.30e-6 9.95e-6 2.60e-5
6.38e-5 1.56e-4 3.47e-4 7.54e-4 1.51e-3 2.28e-3]';
% calculated temperatures for different radius's from the same reference
Pq=[60 75 100 100 150 200 200 300 300 500 500 1000 ...
1000 1000 1500 2000 5000]';
Tq=[2560 2600 2610 2820 2630 2640 2640 ...
2660 2640 2670 2620 2750 2710 2840 2720 2890 2950]';
Uq=120;
Iq=Pq/Uq;
rq=0.5e-6*[45.7 53.3 63.5 61 83.8 96.5 101.6 ...
127 129.5 180.3 185.4 271.8 279.4 289.6 381 457.2 736.6]';
Aq=pi*rq.^2;
%-----
clf;
t = 0;
kk = 0;
while kk < 5000
    c = Iq.^2./Aq;          % T is assumed approximately proportional to I^2/A
    Q = polyfit(c,Tq,1);
    Tc = Q(1)*(P/U)^2/(pi*rc^2)+Q(2); % working temperature at critical spot
    if Tc > Tm
        break
    else
        if rc < 1e-9
            break
        else
            J = interp1q(K,E,Tc);          % Evaporation rate (kg/(m^2 s))
            drc_dt = -J/dw;              % Decrease rate of radius (m/s)
            TC(kk+1,1) = Tc;
            rC(kk+1,1) = rc;
            kk = kk + 1;
            rc = rc + drc_dt*t_step;

            %average values
            T = Q(1)*(P/U)^2/(pi*r^2)+Q(2);
            R = (rho*(1-alfa*(T0-T))*1/(pi*r^2));
            I = U/R;
            P = R*I^2;
            J = interp1q(K,E,T);          % Evaporation rate (kg/m^2 s)
            dr_dt = -J/dw;              % Decrease rate of radius
            TA(kk,1) = T;
            rA(kk,1) = r;
            r = r + dr_dt*t_step;
            t(kk,1) = (kk-1)*t_step/3600;
        end
    end
end
lifetime = t(kk,1)
radius = rc
temp = Tc
plot(t,TC,'+r-',t,TA,'k:');xlabel('time (h)');ylabel('temperature
(K)');title('Filament temperature (K)');
figure

```

```
plot(t,rC,'r-',t,rA,'k:');xlabel('time (h)');ylabel('radius  
(m)');title('Radius of filament(m)');
```