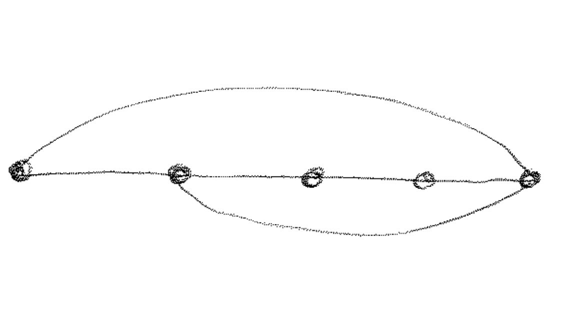
Figure 0.3.2

(i)

(ii)



(iii)

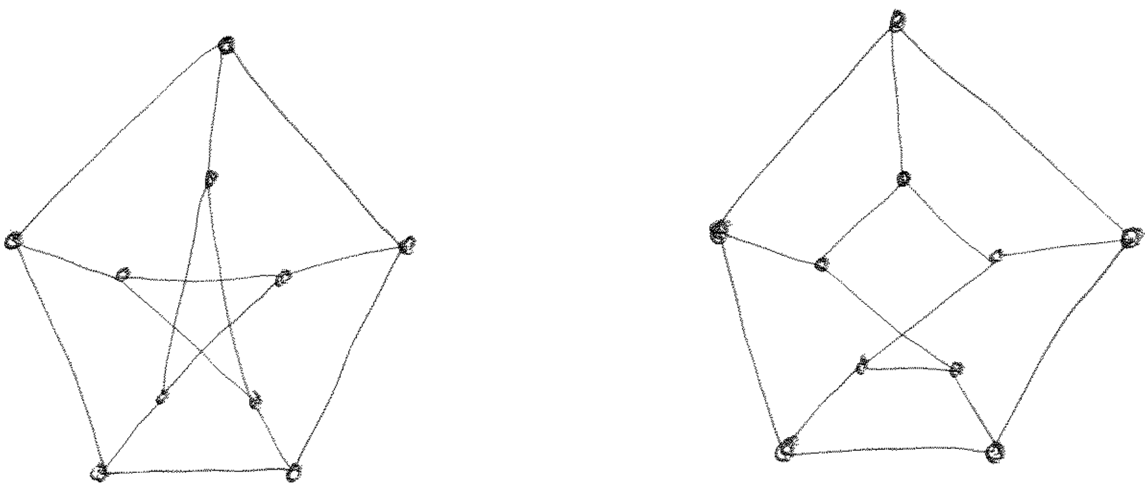
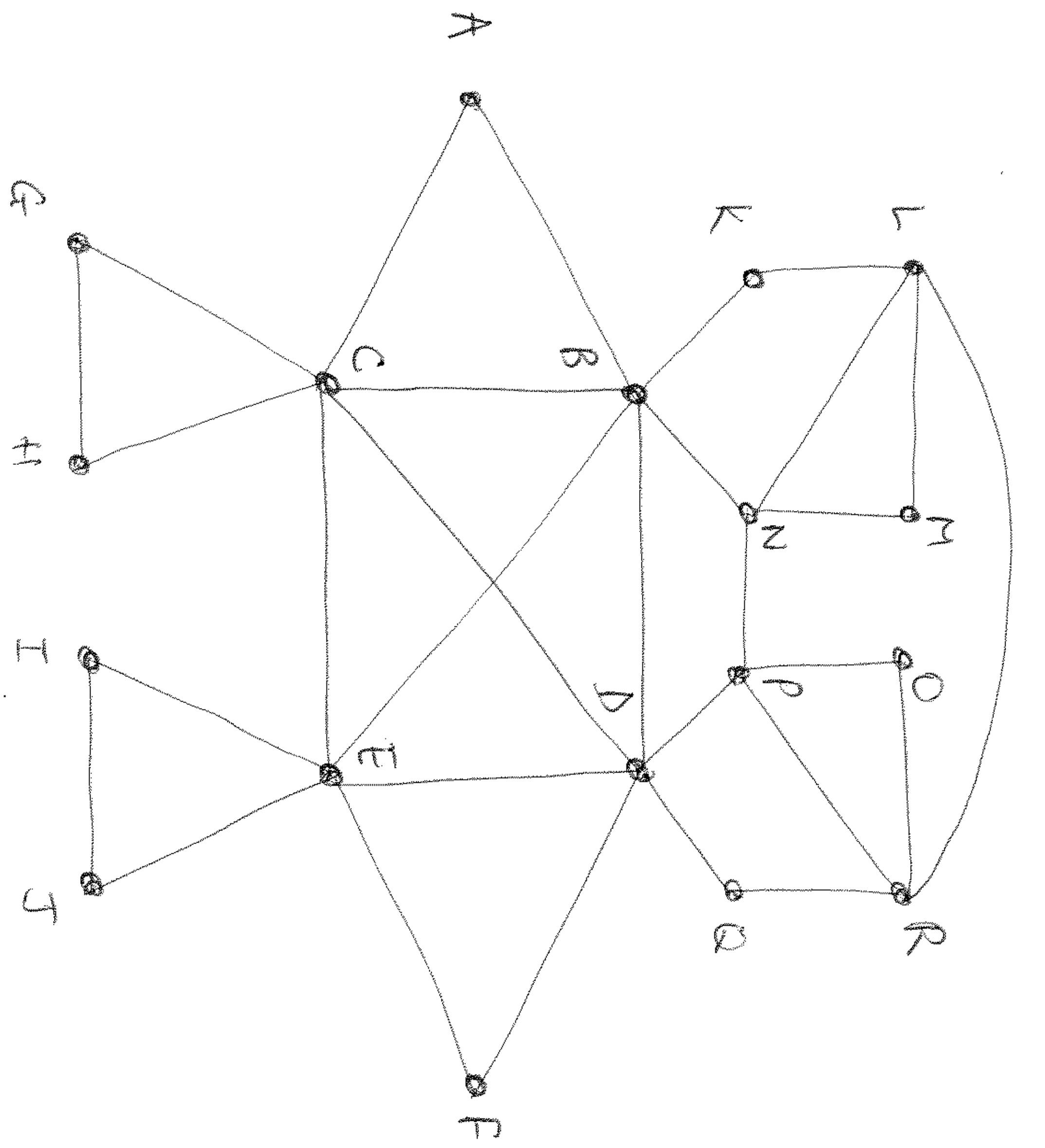


Figure 0.3.3(i)

⇓ look this way



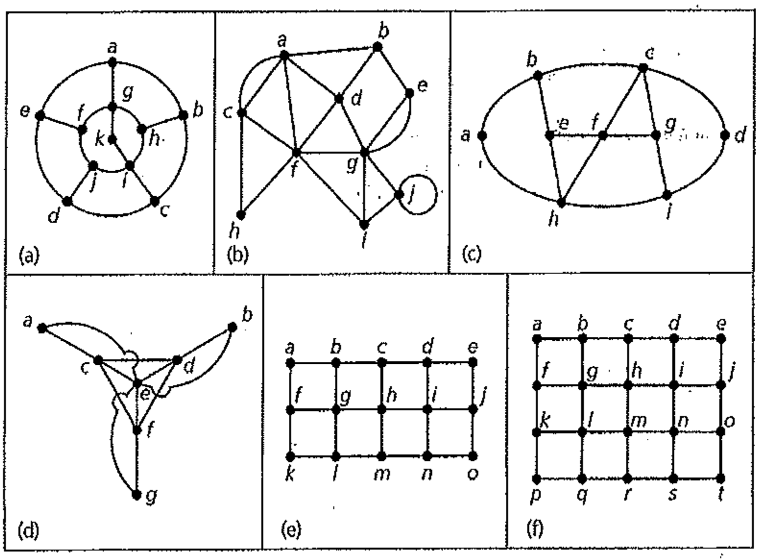


Figure 11.84

5. Consider the graphs in parts (d) and (e) of Fig. 11.84. Is it possible to remove one vertex from each of these graphs so that each of the resulting subgraphs has a Hamilton cycle?

6. If $n \geq 3$, how many different Hamilton cycles are there in the wheel graph W_n? (The graph W_n was defined in Exercise 14 of Section 11.1.)

7. a) For $n \geq 3$, how many different Hamilton cycles are there in the complete graph K_n?

b) How many edge-disjoint Hamilton cycles are there in K_{21}?

c) Nineteen students in a nursery school play a game each day where they hold hands to form a circle. For how many days can they do this with no student holding hands with the same playmate twice?

8. a) For $n \in \mathbf{Z}^+$, $n \geq 2$, show that the number of distinct Hamilton cycles in the graph $K_{n,n}$ is $(1/2)(n-1)!\,n!$

b) How many different Hamilton paths are there for $K_{n,n}$, $n \geq 1$?

9. Let $G = (V, E)$ be a loop-free undirected graph. Prove that if G contains no cycle of odd length, then G is bipartite.

10. a) Let $G = (V, E)$ be a connected bipartite undirected graph with V partitioned as $V_1 \cup V_2$. Prove that if $|V_1| \neq |V_2|$, then G cannot have a Hamilton cycle.

b) Prove that if the graph G in part (a) has a Hamilton path, then $|V_1| - |V_2| = \pm 1$.

c) Give an example of a connected bipartite undirected graph $G = (V, E)$, where V is partitioned as $V_1 \cup V_2$ and $|V_1| = |V_2| - 1$, but G has no Hamilton path.

11. a) Determine all nonisomorphic tournaments with three vertices.

b) Find all of the nonisomorphic tournaments with four vertices. List the in degree and the out degree for each vertex, in each of these tournaments.

12. Prove that for $n \geq 2$, the hypercube Q_n has a Hamilton cycle.

13. Let $T = (V, E)$ be a tournament with $v \in V$ of maximum out degree. If $w \in V$ and $w \neq v$, prove that either $(v, w) \in E$ or there is a vertex y in V where $y \neq v$, w, and (v, y), $(y, w) \in E$. (Such a vertex v is called a *king* for the tournament.)

14. Find a counterexample to the converse of Theorem 11.8.

15. Give an example of a loop-free connected undirected multigraph $G = (V, E)$ such that $|V| = n$ and $\deg(x) + \deg(y) \geq n - 1$ for all $x, y \in V$, but G has no Hamilton path.

16. Prove Corollaries 11.4 and 11.5.

17. Give an example to show that the converse of Corollary 11.5 need not be true.

18. Helen and Dominic invite 10 friends to dinner. In this group of 12 people everyone knows at least 6 others. Prove that the 12 can be seated around a circular table in such a way that each person is acquainted with the persons sitting on either side.

19. Let $G = (V, E)$ be a loop-free undirected graph that is 6-regular. Prove that if $|V| = 11$, then G contains a Hamilton cycle.

20. Let $G = (V, E)$ be a loop-free undirected n-regular graph with $|V| \geq 2n + 2$. Prove that $\overline{G}$ (the complement of G) has a Hamilton cycle.

Figure 0.3.7