Fourier analysis (MMG710/TMA362)

Time: 2009-01-10, 08.30–13.30
Tools: No calculator or handbook is allowed
Questions: Magnus Goffeng, 0762-721860
Grades: Each problem gives 4 points. For MMG710 grades are G (12-17 points) and VG (18-24 points).
For TMA362 grades are 3 (12-14 points), 4 (15-17 points) and 5 (18-24 points).

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1 Use Laplace transform to solve the initial value problem

$$x''(t) - 2x'(t) + x(t) = e^t, \qquad x(0) = 0, \quad x'(0) = 1.$$

2 Find numbers a and b such that the integral

$$\int_0^1 \left| x^3 - ax - b \right|^2 \, dx$$

is minimized.

3 Explain Fourier's method for solving the heat equation for a rod with insulated end-points.

4 (a) Define the notion of convergence in $L^2([0,1])$.

(b) Give an example of a sequence of continuous functions, which converges to 0 in $L^2([0,1])$, but does not converge pointwise to 0 on the whole interval $[0, 1]$.

5 Prove that
$$\sum_{k=0}^{\infty} \frac{1}{(6k+1)(6k+5)} = \frac{\pi\sqrt{3}}{24}.$$

Hint: One way is to expand the constant function $f(x) = 1$ as a Fourier sine series on $0 < x < \pi$, and then put $x = \pi/3$ in the resulting identity.

6 Formulate and prove the inversion formula for Fourier series. If you wish, you may assume that the function you are dealing with is continuous.

Good luck! Hjalmar