Lösning till dugga 3 i MMGF02 Flervariabelanalys, 10 03 02, kl 13.00–13.30.

1. Vi har

$$\begin{aligned}\iint_D 4xy\,dx\,dy &= \int_0^1 2x\Big[y^2\Big]_0^{1+x^2}\,dx = \int_0^1 2x(1+x^2)^2\,dx = \\ &= \Big[\frac{1}{3}(1+x^2)^3\Big]_0^1 = \frac{8}{3}-\frac{1}{3}=\frac{7}{3}\end{aligned}$$

   Svar: 7/3

2. Vi har

$$\begin{aligned}\iiint_K x\,dx\,dy\,dz &= \int_0^1\Big(\int_0^1 x(4-x-y)\,dy\Big)\,dy = \int_0^1\Big(x(4-x)-x/2\Big)\,dx = \\ &= \Big[2x^2-\frac{x^3}{3}-\frac{x^2}{4}\Big]_0^1 = 2-\frac{7}{12}=\frac{17}{12}\end{aligned}$$

   Svar: 17/12

3. Parametrisering ges av $\mathbf{r}(x,y)=(x,y,x^2+y^2)$ med (x,y) i enhetsskivan runt origo (D).

   Vi har

$$\mathbf{r}'_x\times\mathbf{r}'_y = \left\{\begin{matrix}1 & 0 & 2x\\ 0 & 1 & 2y\end{matrix}\right\} = (-2x,-2y,1)$$

   så $|\mathbf{r}'_x\times\mathbf{r}'_y| = \sqrt{4x^2+4y^2+1}$.

   Arean av ytan ges av

$$\begin{aligned}\int_Y dS &= \int_D \sqrt{4x^2+4y^2+1}\,dx\,dy = \{\text{ polära koord.}\} = \\ &= \int_0^1 r\sqrt{4r^2+1}\Big(\int_0^{2\pi}dt\Big)\,dx = \frac{2\pi}{12}\Big[(4r^2+1)^{3/2}\Big]_0^1 = \frac{\pi}{6}(5\sqrt{5}-1)\end{aligned}$$

   Svar: $\pi(5\sqrt{5}-1)/6$