Joachim Hilgert: On the Lewis functional equation

Abstract: The Lewis functional equation $\phi(z) = \phi(z+1) + \lambda z^{-2s}\phi\left(1 + \frac{1}{z}\right)$ with parameters $\lambda = \pm 1$ and $s \in \mathbb{C}$ characterizes (together with some growth conditions) the image of the Maa wave forms for $PSL(2, \mathbb{Z})$ as the space of period functions under an analogue of the Eichler-Manin-Shimura correspondence established by Lewis and Zagier. At the same time it is satisfied by the eigenfunctions of the Ruelle transfer operator associated with the geodesic flow on the modular surface. Similar relations are expected to hold for more general (congruence) subgroups of $PSL(2, \mathbb{Z})$.

In this talk we will descibe the (generalized) Lewis equations for the groups $\Gamma_0(n)$ and show how to construct solutions. The construction is based on continued fractions and shows a striking analogy with part of the Atkin-Lehner theory of old and new (modular) forms.