Andreas Juhl: Families of conformally invariant differential operators.

Abstract:

Let M be a smooth manifold. A conformally invariant differential operator is a rule which associates a natural differential operator $D(g) : C^\infty(M)- > C^\infty(M)$ to any Riemannian metric g on M so that $e^{a\varphi}D(g)e^{b\varphi} = D(e^{2\varphi}g)$ for certain conformal weights a, b and any function $\varphi \in C^\infty(M)$. There are analogous versions for operators on sections of vector bundles. On the standard conformally flat sphere such operators are in bijection to homomorphism of generalized Verma modules for the Lie algebra of the conformal group of the sphere. Via Cartan's normal connection many of these (but not all) homomorphism induce conformally invariant operators on any conformal manifold. The best known special case is the Yamabe operator (which plays a central role in the study of the scalar curvature).

In the talk we adress the problem to study analogous questions for local operators from smooth functions on a conformal manifold to smooth functions on a submanifold. In the conformally flat case such operators again are induced by certain homomorphisms of Verma modules. It is well-known that non-trivial homomorphisms between 2 Verma modules of a semisimple Lie algebra exist only for a discrete set of conformal weights. Here we have to consider homomorphisms between Verma modules for 2 different Lie algebras and we have constructed polynomial families of such homomorphisms. Moreover, these are related to classical orthogonal polynomials and satisfy simple recursive relations. The structure of these homomorphism is related to the asymptotics of eigenfunctions of the Laplacian on hyperbolic manifolds (scattering theory). We describe the induced operators in the conformally flat situation of spheres. It is a challenging problem to investigate the curved analogs since these lead to a series of conformal invariants of which Branson's Q-curvature is a special case. Finally, we describe some applications to automorphic distributions (for Kleinian groups).