Quantum resonances for transparent obstacles

We study the distribution of resonances for the transmission problem for a strictly convex bounded obstacle with a smooth boundary in $\mathbf{R}^n$, $n \geq 2$. If the speed of propagation in the interior of the body is strictly less than that in the exterior, we obtain an infinite sequence of resonances tending rapidly to the real axis. These resonances are associated with a quasimode for the transmission problem the frequency support of which coincides with the corresponding gliding manifold $\mathcal{K}$.

If the speed of propagation in the interior is bigger than that in the exterior, we prove that there exists a strip in the upper half plane containing the real axis, which is free of resonances. We also obtain a uniform decay of the local energy for the corresponding mixed problem with an exponential rate of decay when the dimension is odd, and polynomial otherwise. It is well known that such a decay of the local energy holds for the wave equation with Dirichlet (Neumann) boundary conditions for any nontrapping obstacle. In our case, however, $\mathcal{O}$ is a trapping obstacle for the corresponding classical system.

We show also that there is a free of resonances region in the complex upper half plane given by $\{C \leq \operatorname{Im}\lambda \leq C_1|\lambda|^{1/3} - C_2\}$, where C, C_1 and C_2 are positive constants. Moreover, we obtain asymptotics for the number of resonances counted with multiplicities in the region $\{0 < \operatorname{Im}\lambda \leq C,\ 0 < \operatorname{Re}\lambda \leq r\}$ as $r \to \infty$, where $C > 0$ is the same constant as above.