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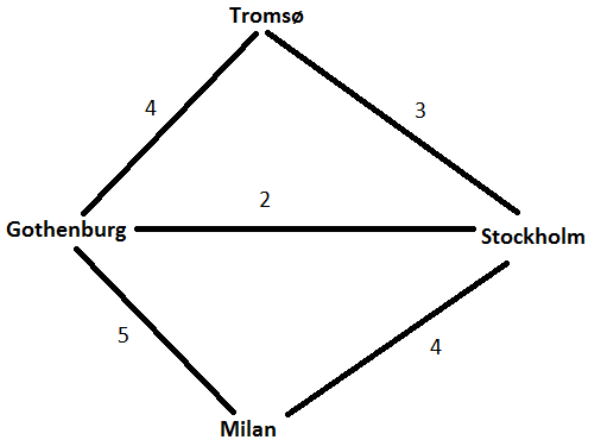
Applied Mathematics and Statistics
Chalmers and GU

MVE051 / MSG810 Matematisk statistik och diskret matematik

Exam 13 January 2017, 8:30 - 12:30

Allowed aids: Chalmers-approved calculator
and one (two-sided) A4 sheet of paper with your own notes.
Total number of points: 30. To pass, at least 12 points are needed.
Note: All answers should be motivated.

1. **(5 points)** Consider the graph in the figure below, where each edge in the graph represents a daily flight from one city to another and the number close to it represents how long the flight takes in hours. For example, one can take a direct flight from Milan to Stockholm and arrive at the destination in 4 hours. Marco would like to reach Tromsø from Milan. Let $X_n, n \in \mathbb{N}$ denote the city in which Marco is at day n, so $X_0 =$ "$Milan$". For each day and each city, all flights out of that city except one are cancelled because of bad weather; we assume the flight not cancelled is randomly selected with equal probability among the available flights. Marco will always take the available flight, except if he reaches Tromsø: Then he will stay there forever.



   (a) Is $\{X_n\}_{n\in\mathbb{N}}$ a Markov chain? If so, write down its transition matrix.

   (b) Compute the expected number of days it takes for Marco to reach Tromsø.

   (c) Let Y_n denote the total number of hours Marco has been in the air since leaving Milan. Is $\{Y_n\}_{n\in\mathbb{N}}$ a Markov chain? Motivate.

2. **(5 points)** An experiment whose outcome can be either "Success" or "Fail" is repeated independently 10 times. Let X be the random variable that counts the number of successes in the 10 trials:

   (a) Assume that the probability that the i-th experiment will be a success is $p_i = \frac{i}{10}$. Does X have binomial distribution? If so, specify the parameters. Motivate your answer in any case.

   (b) Assume that the probability that the i-th experiment will be a success is $p_i = \frac{i}{10}$. Find $P(X = 1)$.

   (c) Assume that the probability that the i-th experiment will be a success is $p_i = \frac{1}{10}$. Does X have binomial distribution? If so, specify the parameters. Motivate your answer in any case.

   (d) Suppose we have an urn with 10 balls. On 8 of them it is written "Success" and on the remaining 2 "Fail". We extract 5 balls from the urn and let Y count the number of "Success" balls picked. Does Y have binomial distribution? If so, specify the parameters. Motivate your answer in any case.

3. **(5 points)** At Chalmers University of Technology 200 students have taken part in a Mathematics course. Suppose that each student has a probability of 70% of passing an exam in such a course.

   (a) Compute the probability that a student will pass the exam in less than 3 attempts, by assumining that the attempts are independent.

   (b) What is the distribution of the random variable that models the number of attempts done by a student to pass the exam? What are its parameters? (Assume that a student cannot take an exam he has already passed, but can attempt the exam as many times as he wants.)

   (c) Assume that at the first exam 200 students take the exam. Assume moreover that the students cannot withdraw from the exam (either they pass or they fail). Let X denote the number of persons that will pass this exam and Y denote how many will fail it. Find the correlation coefficient between X and Y.

4. **(5 points)** A rare disease has over many years been observed with an average of 8 new cases in Sweden each year. Not much is known about the causes of the disease, but it is assumed that cases occur independently.

   (a) What is the probability of 3 or fewer cases during one year?

   (b) What is the probabilty of at least one case during one month (assuming 12 months of equal lengths)?

   (c) Given that a new case has just been observed, what is the probability that no new cases are observed for the next 2.5 months?

(d) Doctor X is wondering whether the occurrence of the disease is actually increasing. During the last decade, there have been 103 new cases in Sweden. What is the (approximate) probability of observing 103 or more cases assuming that there has been no increase in the occurrence?

5. **(5 points)** In a random sample of 43 people working at company X, 37 said they were satisfied with their job, while in a similar sample of size 35 from company Y, 21 said they were satistifed.

   (a) Give an estimate of how many more percent of the workers are satisfied with their job at X compared to at Y.

   (b) Compute a 90% confidence interval for the estimate you made in (a)

   (c) Assume you want to pose the same question to n persons working at company A and n persons working at B. Assume you want to be sure that the resulting 90% confidence interval has length at most 10%. How big does n need to be?

6. **(5 points)** Let A and B be two events such that $P(A) > 0, P(B) > 0$.

   (a) Are each of the following relationships necessarily true, necessarily false, or it may be either true or false? (Motivate your answer)

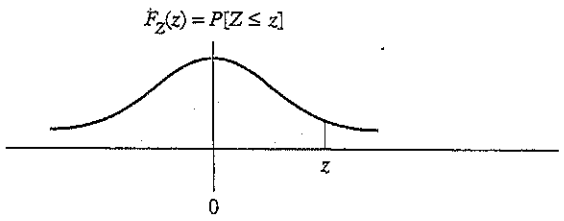
   - $P(A) < P(A|B)$;
   - $P(A) = P(A|B)$;

   (b) Suppose now that $P(A|B) > P(A)$. Are each of the following relationships necessarily true, necessarily false, or it may be true or false? (Motivate your answer)

   - $P(A \cap B) = 0$;
   - $P(B|A) > P(B)$;

TABLE V

Cumulative distribution: Standard normal



$F_Z(z) = P[Z \le z]$

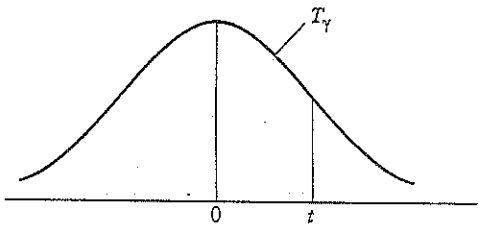
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1921	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

TABLE V

Cumulative distribution: Standard normal *(concluded)*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

T distribution



Column heading = cumulative probability

Row heading = degrees of freedom

Row ∞ = standard normal values

$P[T_\gamma \leq t]$

γ	.6	.75	.9	.95	.975	.99	.995	.999	.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.317	636.607
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.611	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
31	0.256	0.682	1.309	1.696	2.040	2.453	2.744	3.375	3.633
32	0.255	0.682	1.309	1.694	2.037	2.449	2.738	3.365	3.622
33	0.255	0.682	1.308	1.692	2.035	2.445	2.733	3.356	3.611
34	0.255	0.682	1.307	1.691	2.032	2.441	2.728	3.348	3.601
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
36	0.255	0.681	1.306	1.688	2.028	2.434	2.719	3.333	3.582
37	0.255	0.681	1.305	1.687	2.026	2.431	2.715	3.326	3.574
38	0.255	0.681	1.304	1.686	2.024	2.429	2.712	3.319	3.566
39	0.255	0.681	1.304	1.685	2.023	2.426	2.708	3.313	3.558
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
41	0.255	0.681	1.303	1.683	2.020	2.421	2.701	3.301	3.544
42	0.255	0.680	1.302	1.682	2.018	2.418	2.698	3.296	3.538
43	0.255	0.680	1.302	1.681	2.017	2.416	2.695	3.291	3.532
44	0.255	0.680	1.301	1.680	2.015	2.414	2.692	3.286	3.526

TABLE VI

T distribution *(concluded)*

γ	.6	.75	.9	.95	.975	.99	.995	.999	.9995
45	0.255	0.680	1.301	1.679	2.014	2.412	2.690	3.281	3.520
46	0.255	0.680	1.300	1.679	2.013	2.410	2.687	3.277	3.515
47	0.255	0.680	1.300	1.678	2.012	2.408	2.685	3.273	3.510
48	0.255	0.680	1.299	1.677	2.011	2.407	2.682	3.269	3.505
49	0.255	0.680	1.299	1.677	2.010	2.405	2.680	3.265	3.500
50	0.255	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
51	0.255	0.679	1.298	1.675	2.008	2.402	2.676	3.258	3.492
52	0.255	0.679	1.298	1.675	2.007	2.400	2.674	3.255	3.488
53	0.255	0.679	1.298	1.674	2.006	2.399	2.672	3.251	3.484
54	0.255	0.679	1.297	1.674	2.005	2.397	2.670	3.248	3.480
55	0.255	0.679	1.297	1.673	2.004	2.396	2.668	3.245	3.476
56	0.255	0.679	1.297	1.673	2.003	2.395	2.667	3.242	3.473
57	0.255	0.679	1.297	1.672	2.002	2.394	2.665	3.239	3.470
58	0.255	0.679	1.296	1.672	2.002	2.392	2.663	3.237	3.466
59	0.254	0.679	1.296	1.671	2.001	2.391	2.662	3.234	3.463
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
61	0.254	0.679	1.296	1.670	2.000	2.389	2.659	3.229	3.457
62	0.254	0.678	1.295	1.670	1.999	2.388	2.658	3.227	3.455
63	0.254	0.678	1.295	1.669	1.998	2.387	2.656	3.225	3.452
64	0.254	0.678	1.295	1.669	1.998	2.386	2.655	3.223	3.449
65	0.254	0.678	1.295	1.669	1.997	2.385	2.654	3.221	3.447
66	0.254	0.678	1.295	1.668	1.997	2.384	2.652	3.218	3.444
67	0.254	0.678	1.294	1.668	1.996	2.383	2.651	3.217	3.442
68	0.254	0.678	1.294	1.668	1.995	2.382	2.650	3.215	3.440
69	0.254	0.678	1.294	1.667	1.995	2.382	2.649	3.213	3.437
70	0.254	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435
71	0.254	0.678	1.294	1.667	1.994	2.380	2.647	3.209	3.433
72	0.254	0.678	1.293	1.666	1.993	2.379	2.646	3.207	3.431
73	0.254	0.678	1.293	1.666	1.993	2.379	2.645	3.206	3.429
74	0.254	0.678	1.293	1.666	1.993	2.378	2.644	3.204	3.427
75	0.254	0.678	1.293	1.665	1.992	2.377	2.643	3.203	3.425
76	0.254	0.678	1.293	1.665	1.992	2.376	2.642	3.201	3.423
77	0.254	0.678	1.293	1.665	1.991	2.376	2.641	3.200	3.422
78	0.254	0.678	1.292	1.665	1.991	2.375	2.640	3.198	3.420
79	0.254	0.678	1.292	1.664	1.990	2.375	2.640	3.197	3.418
80	0.254	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416
81	0.254	0.678	1.292	1.664	1.990	2.373	2.638	3.194	3.415
82	0.254	0.677	1.292	1.664	1.989	2.373	2.637	3.193	3.413
83	0.254	0.677	1.292	1.663	1.989	2.372	2.636	3.191	3.412
84	0.254	0.677	1.292	1.663	1.989	2.372	2.636	3.190	3.410
85	0.254	0.677	1.292	1.663	1.988	2.371	2.635	3.189	3.409
86	0.254	0.677	1.291	1.663	1.988	2.371	2.634	3.188	3.407
87	0.254	0.677	1.291	1.663	1.988	2.370	2.634	3.187	3.406
88	0.254	0.677	1.291	1.662	1.987	2.369	2.633	3.186	3.405
89	0.254	0.677	1.291	1.662	1.987	2.369	2.632	3.184	3.403
90	0.254	0.677	1.291	1.662	1.987	2.369	2.632	3.183	3.402
91	0.254	0.677	1.291	1.662	1.986	2.368	2.631	3.182	3.401
92	0.254	0.677	1.291	1.662	1.986	2.368	2.630	3.181	3.400
93	0.254	0.677	1.291	1.661	1.986	2.367	2.630	3.180	3.398
94	0.254	0.677	1.291	1.661	1.986	2.367	2.629	3.179	3.397
95	0.254	0.677	1.291	1.661	1.985	2.366	2.629	3.178	3.396
96	0.254	0.677	1.290	1.661	1.985	2.366	2.628	3.177	3.395
97	0.254	0.677	1.290	1.661	1.985	2.365	2.627	3.176	3.394
98	0.254	0.677	1.290	1.661	1.984	2.365	2.627	3.176	3.393
99	0.254	0.677	1.290	1.660	1.984	2.365	2.626	3.175	3.392
100	0.254	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.391
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291