Lösningar till tentamen i Matematisk statistik och diskret matematik D2 (MVE055/MSG810).
Den 11 januari 2012. These are sketches of the solutions.

1. Lösning:
   as $h(x), g(x) \geq 0$ and $\alpha \in [0,1]$ then $d(x) \geq 0$ and now we check whether $d(x)$ integrates to 1
   $\int_{-\infty}^{\infty} d(x)\mathrm{d}x = \int_{-\infty}^{\infty} \alpha g(x) + (1-\alpha)h(x)\mathrm{d}x = \alpha \int_{-\infty}^{\infty} g(x)\mathrm{d}x + (1-\alpha)\int_{-\infty}^{\infty} h(x)\mathrm{d}x =$
   $\alpha + 1 - \alpha = 1$

2. Lösning:
   a) $\sum_{i=1}^{n} w_i = 1$
   b) $\sigma^1 \sum_{i=1}^{n} w_i^2$

3. Lösning:
   a) Geometric distribution, $P(K = k) = (1-p)^k p$ for $k = 0, 1, 2, \ldots$
   b) $P(X = 2^k) = (1-p)^{k-1} p$ for $k = 0, 1, 2, \ldots$
   c) $\mathrm{E}[K] = 1/p$. If $p > 0.5$ $\mathrm{E}[X] = 2p(2-2p)/(2p-1)$ and if $p \leq 0.5$ $\mathrm{E}[X] = \infty$. This means that if our chances of winning are lesser than 0.5 even though we expect to win the game we need an infinite amount of money to do this.

4. Lösning:
   a) $M_X(t) = \mathcal{E}[e^{tX}]$, it is the generating function of the sequance of the moments of the random variable
   b) $pe^t/(1-(1-p)e^t)$ or $(1-p)e^t/(1-pe^t)$ depending what you took for p
   c) $\mathrm{E}[X] = 1/p$ (or $1/(1-p)$), $\mathrm{Var}[X] = (1-p)/p^2$ (or $p/(1-p)^2$)