Marco Longfils and Petter Mostad
Applied Mathematics and Statistics
Chalmers and GU

MVE055 / MSG810 Matematisk statistik och diskret matematik

Exam 30 October 2018, 8:30 - 12:30

Allowed aids: Chalmers-approved calculator
and one (two-sided) A4 sheet of paper with your own notes.
Total number of points: 30. To pass, at least 12 points are needed.
Note: All answers should be motivated.

1. **(5 points)** Let $X_1, ..., X_n$ be independent and identically distributed random variables with normal distribution with mean μ and variance σ^2. Consider the following estimators for the mean μ:

$$E_1 = \frac{1}{2}\left[X_1 + \frac{3}{4}X_2 + \frac{X_3}{4}\right], \quad E_2 = \left(X_1 - \frac{1}{2}\right)^2 - \frac{1}{4}, \quad E_3 = \frac{\sum_{i=1}^{3} iX_i}{6}.$$

   (a) Which of the above estimators is unbiased for μ?

   (b) Among the unbiased estimators found in task a), which one has the minimum variance?

2. **(5 points)** One package of potatoes contains 10 potatoes and weighs exactly 500 grams. Denote by $X_1, ..., X_{10}$ the weights of each potato.

   (a) Are the random variables $X_1, ..., X_{10}$ independent? Compute the correlation coefficient of $\rho(X, Y)$ where $X = X_1$ and $Y = \sum_{i=2}^{10} X_i$.

   (b) Define for $i = 1, ..., 10$
   $$Z_i = \begin{cases} 1, X_i > 50\text{grams} \\ 0, \text{otherwise} \end{cases}$$
   Let $Z = \sum_{i=1}^{10} Z_i$. Is the distribution of Z binomial? Motivate.

3. **(5 points)** Johan has three email accounts: gmail, Chalmers, and GU. 40% of all the emails Johan receives come into his gmail account, 35% come into his Chalmers account, and the remaining 25% into his GU account. Unfortunately, Johan often receives spam messages. In particular, 2% of all the messages into the gmail account are spam, whereas 1% and 5% respectively of the message into the Chalmers and GU accounts are spam.

   (a) What is the probability that a randomly selected email is spam?

   (b) What is the probability that a message came into the gmail account given it is spam?

4. **(5 points)** Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined by the recursion

$$\begin{cases} a_0 = 5 \\ a_n = 3a_{n-1} + 2, \quad n \geq 1. \end{cases}$$

Find a_n.

5. **(5 points)** In a sample of $n = 541$ people 120 were obese according to their body mass index following the World Health Organization recommendations. We would like to check if there is compelling evidence that more than 20% of the individuals in the population are obese.

   (a) State and perform an appropriate hypothesis test at level $\alpha = 0.05$.

   (b) Explain Type I and Type II errors in the context of this problem. Given your conclusion in a), what type of error could possibly occur in this situation?

6. **(5 points)** Alice and Bob play the following game: they repeatedly extract cards from a standard deck with 52 cards, and after a card has been drawn they put it back in the deck. Alice wins if they draw two red cards (diamons or hearts) in succession, while Bob wins as soon as a Spades is drawn. The game ends when either Alice or Bob wins. What is the probability that Alice wins?