MVE055 2017 Lecture 15

Marco Longfils

Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg

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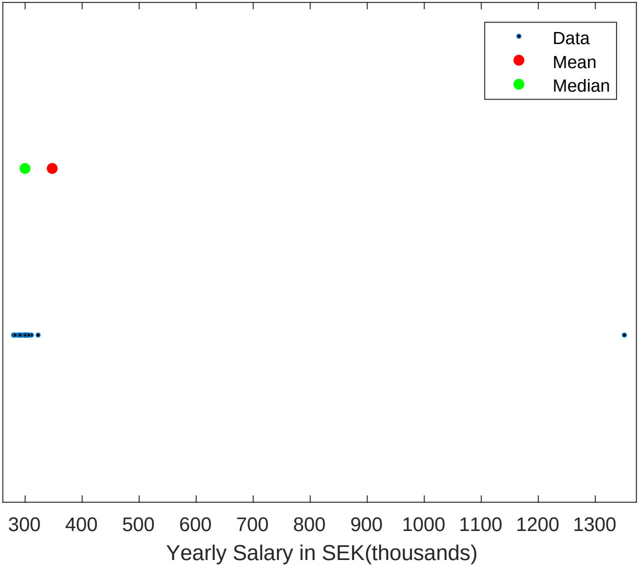
Parametric vs. Nonparametric

- The data have to be numeric:
- We need/can make assumptions on the underlying distribution of the data (e.g. normality).
- They often have higher statistical power than corresponding nonparametric test. That is, When a significant effect exists, parametric test are more likely to detect this effects.
- If the sample size is large "enough", they can still perform well even in the case where assumptions are not met (e.g. T test with equal variances, nonnormal data)

Nonparametric test

- The data can be numeric, ordinal, or nominal.
- We do not specify the underlying distribution of the data ("distribution-free").
- Usable even with small samples;
- T-test (parametric) is testing equality of means, where sometimes the mean might not be the "best" measure of location.


Data
Mean
Median
300
400
500
600
700
800
900
1000
1100
1200
1300
Yearly Salary in SEK(thousands)

Wilcoxon Rank-Sum test

- Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent random samples of, respectively, X and Y.
- We want to test if X and Y have the same distribution, i.e.

$$H_0: \text{ X and Y have the same distribution}$$

$$H_1: \text{ X and Y do not have the same distribution}$$

- Under some assumptions, Wilcoxon test can be interpreted as a test on medians, i.e. denote by M_X, M_Y the medians

$$H_0: M_X = M_Y$$

$$H_1: M_X \neq M_Y$$

- Similarly, one can consider one-sided tests.

An example

- Suppose we have the following data regarding the height of individuals (in centimeters):

Male	Female
180	175
193	168
178	163
188	165
185	173
170	
183	

- We would like to investigate if the distribution of the height is the same in this two groups, but (for one of the reason specified above) we cannot use a T-test.

An example

- Sort the data in ascending order: (Men,Women)

163 165 168 170 173 175 178 180 183 185 188 193

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- Rank the values starting from 1 for the lowest value:

Height	163	165	168	170	173	175	178	180	183	185	188	193
Rank	1	2	3	4	5	6	7	8	9	10	11	12

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- Define W_5 to be the sum of the ranks for women, i.e.

$$W_5 = 1 + 2 + 3 + 5 + 6 = 17$$

- We look in the Statistical table for the critical values for Wilcoxon Rank Sum test with $m = 5$ and $n = 7 = m + 2$

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- Define W_5 to be the sum of the ranks for women, i.e.

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- We look in the Statistical table for the critical values for Wilcoxon Rank Sum test with $m = 5$ and $n = 7 = m + 2$
- We reject H_0 at a 5% significance level if $W_5 < 20$ or $W_m > 45$. We conclude: men are taller than women.

Wilcoxon Rank-Sum test

- Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent random samples of, respectively, X and Y. Assume $m \leq n$.
- Sort the values of $X_1, ..., X_m, Y_1, ..., Y_n$ in ascending order and rank them starting from 1 up to $m + n$.
- If there are ties, all the tied values get the average rank of the tied value, for example if we have three tied observation with height 180 and ranks $i, i+1, i+2$ then each of those observation get the rank
$$\frac{i + (i+1) + (i+2)}{3}$$
- Compute W_m, the sum of the ranks of the X's;
- Look at the statistical tables (Table X in (MA)) for the critical values;

Wilcoxon Signed Rank test

- Is a nonparametric test for testing equality of distribution of two **DEPENDENT** samples (null and alternative hypothesis as before);
- Example: we would like to test the effect of a new drug on blood pressure. We measure the (systolic) blood pressure of 7 patients before and after they take the drug.

Before	After
120	115
113	108
118	113
128	125
115	123
110	112
123	122

- We can see that the measurements of the blood pressure are naturally paired as "before" (Xs) and "after" (Ys) $(X_1, Y_1), ..., (X_7, Y_7)$;

Wilcoxon Signed Rank test

- We compute the pair differences $X_i - Y_i,\ i = 1, ..., 7$.

X	Y	|X-Y|	sign(X-Y)	rank	signed rank
120	115	5	$+$	5	$+5$
113	108	5	$+$	5	$+5$
118	113	5	$+$	5	$+5$
128	125	3	$+$	3	$+3$
115	123	8	-	7	-7
110	112	2	-	2	-2
123	122	1	$+$	1	$+1$

- Define

$$W_+ = \sum_{\text{all positive ranks } R_i} R_i, \qquad |W_-| = \sum_{\text{all negative ranks } R_i} |R_i|$$

- zero differences are assigned the sign that is "least leading" to the rejection of the null hypothesis;

Wilcoxon Signed Rank test

- In our example
$$W_+ = 19, \quad |W_-| = 9$$
- For a two sided test, the statistic to be used is $\min\{W_+, |W_-|\}$; for right tailed test ("X>Y") we use $|W_-|$, for left tailed test ("X<Y") W_+;
- In any case we reject the null hypothesis if the test statistic is smaller than the critical value found in Table VIII of (MA);
- We perform the two tailed test at the level $\alpha = 0.05$. The critical point is 2 and $\min\{W_+, |W_-|\} = 9 > 2$. Thus we do not reject the null hypothesis.

Wilcoxon Signed Rank-Sum test

- Let $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ be two dependent (i.e. paired) random samples of, respectively, X and Y.
- Sort the absolute values of $X_1 - Y_1, ..., X_n - Y_n$ in ascending order and rank them starting from 1 up to n.
- If there are ties, all the tied values get the average rank of the tied value;
- Compute the signed ranks by multiplying the sign of the difference $X_i - Y_i$ by its rank;
- Compute $\{W_+, |W_-|\}$
- Look at the statistical tables (Table VIII in (MA)) for the critical value;
- Reject H_0 if the test statistic is smaller than the critical value