MVE055 2018 Lecture 5

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More examples of distributions

- We have seen three examples of distributions: Binomial, Geometric, and Normal.
- These are families of distributions, as for each choice of their parameters corresponds a specific distribution.
- Other examples of such families are: Poisson, Hypergeometric, negative Binomial, Gamma, Chi-squared, and exponential. Each of these families can be used to model different phenomena, depending on the properties characterizing the phenomenon.

Poisson distribution

Definition

A discrete random variable X with possible values $x = 0, 1, 2, ...$ is said to have Poisson distribution with parameter $\lambda > 0$ if it has density function

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

- $\mathbb{E}[X] = \lambda$, $\mathrm{Var}(X) = \lambda$.
- λ is the average number of "events" in 1 unit of time.
- If $X_1 \sim Poisson(\lambda_1)$, $X_2 \sim Poisson(\lambda_2)$, and they are independent we have $X_1 + X_2 \sim Poisson(\lambda_1 + \lambda_2)$

Hypergeometric distribution

Definition

A discrete random variable X is said to have Hypergeometric distribution with parameters $N, n, r \in \mathbb{N}$ if it has density function

$$f(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}, \qquad \max\{0, n-(N-r)\} \leq x \leq \min\{n, r\}$$

- $\mathbb{E}[X] = n\frac{r}{N}$, $\mathrm{Var}(X) = n\frac{r}{N}\frac{N-r}{N}\frac{N-n}{N-1}$.
- We select n objects from N objects, of which r has a trait. X counts how many of the selected objects have the trait.

negative Binomial distribution

Definition

A discrete random variable X is said to have negative Binomial distribution with parameters $p \in [0,1], r \in \mathbb{N} \setminus \{0\}$ if it has density function

$$f(x) = \binom{x-1}{r-1}(1-p)^{x-r}p^r, \qquad x = r, r+1, r+2, ...$$

- $\mathbb{E}[X] = \frac{r}{p}$, $\text{Var}(X) = \frac{r(1-p)}{p^2}$.
- Consider a sequence of independent and identical experiments, each one with probability p of success. X models the number of trails needed to obtain r successes.

Gamma distribution

Definition

A continuous random variable X is said to have Gamma distribution with parameters $\alpha > 0, \beta > 0$ if its density function is

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}}, \qquad x > 0$$

where $\Gamma(\alpha) = \int_0^{\infty} z^{\alpha-1} e^{-z}\, dz$.

- $\mathbb{E}[X] = \alpha\beta$, $\mathrm{Var}(X) = \alpha\beta^2$.
- Sometimes it is parametrized with $\beta' = \frac{1}{\beta}$.
- Notation: $X \sim Gamma(\alpha, \beta)$.

Chi-Squared distribution

Definition

A continuous random variable X is said to have Chi-squared distribution with γ degrees of freedom if it has Gamma distribution with parameters $\alpha = \gamma/2, \beta = 2$

$$\chi^2(\gamma) = Gamma(\gamma/2, 2)$$

- $\mathbb{E}[X] = \gamma$, $\text{Var}(X) = 2\gamma$.
- If $Z_1, ..., Z_k$ are independent random variables with standard normal distribution, then

$$Z_1^2 + \cdots + Z_k^2 \sim \chi^2(k)$$

Exponential distribution

Definition

A continuous random variable X is said to have exponential distribution with parameter $\beta > 0$ if its density function is

$$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \qquad x > 0$$

- $\mathbb{E}[X] = \beta$, $\mathrm{Var}(X) = \beta^2$.
- Sometimes it is parametrized with $\lambda = \frac{1}{\beta}$.
- $F_X(x) = 1 - e^{-\frac{x}{\beta}}$.
- $Exponential(\beta) = Gamma(1, \beta)$.