RANDOM PROCESSES WITH APPLICATIONS 2007

Optional home work 2

Day assigned: **September 30**

Assignment deadline: **11:45 am, October 12**

Problem 1. Consider two random variables defined as linear combinations of other random variables,

$$Y = \sum_1^n a_i Y_i, \quad Z = \sum_1^m b_j Z_j$$

.

(a) Show that

$$Cov(Y, Z) = \sum_i \sum_j a_i b_j Cov(Y_i, Z_j)(1)$$

(1)

Next, let $X_1, X_2, \ldots X_n$ be independent observations on the random variable $X \sim N(\mu, \sigma^2)$. Consider the sample mean and variance

$$\hat{\mu} = \frac{1}{n}\sum_1^n X_i, \quad s^2 = \frac{1}{n-1}\sum_1^n (X_i - \hat{\mu})^2.$$

(b) Use (a) to show that the sample mean $\hat{\mu}$ and $X_i - \hat{\mu}$ are uncorrelated, $i = 1, 2, \ldots, n$. (1)

(c) Use (b) to show that the sample mean and the sample variance are independent. (1)

Problem 2. Consider the rectangular pulse function $g(t) = u(t) - u(t-1)$ and the random variable $T \sim U(0, 1)$. Define the random process

$$Y(t) = g(t - T)$$

(a) Find the CDF of $Y(t)$. (1)

(b) Find the mean function $\mu_Y(t)$. (1)

(c) Find the autocovariance function $C_{YY}(t_1, t_2)$. (1)

Problem 3. Let $X(t)$ be a white sense stationary random process with $\mu_X(t) = 0$ that is ergodic in the mean and the autocorrelation, and let $Y(t) = ZX(t)$, where Z is a random variable with expected value zero which is independent of $X(t)$. Answer and explain:

(a) Is the process $Y(t)$ ergodic in mean? (1)

(b) Is it ergodic in autocorrelation? (1)

Problem 4. A shot noise process with random amplitude is defined by

$$X(t) = \sum_{1}^{\infty} A_i h(t - s_j),$$

where the S_j are the points of occurrences of a Poisson process $N(t)$ of rate λ, and A_i are iid random variables independent of $N(t)$.

(a) Compute $\mu_X(t)$. (2)

(b) Compute $C_{XX}(t_1, t_2)$. (2)

Problem 5. Consider the second order autoregressive process defined by

$$Y_n = \frac{3}{4}Y_{n-1} - \frac{1}{8}Y_{n-2} + W_n,$$

where W_n is the zero mean white noise process.

(a) Show that the unit impulse response of the linear system producing Y is

$$h_n = 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n, \; n \geq 0.$$

(1)

(b) Find the transfer function of the system. (1)

(c) Find the PSD of the process and its autocorrelation function. (1)