Written test for examination in MVE135
Random processes with applications, 2008-08-18, 08:30 - 12:30, house V.
On duty: Rossitza Dodunekova, 772 3534.
Time of visits 10:00 and 11:30.
Allowed material: The handbook *Beta*, Collection of Formulas for MVE135, calculators approved by Chalmers.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. In pulse amplitude modulation (PAM), a PAM word consists of a sequence of pulses, where each pulse may take on a given number of amplitude levels. If each PAM word, four pulses long, is equally likely to occur and each pulse can have one of three levels $\{0, 1, 2\}$, what is the probability of a PAM word occurring with exactly two pulses of level 1? 3p

Problem 2. X and Y are independent Gaussian random variables with expectation 0 and variance 1.

(a) Evaluate $P(X < Y)$. 3p

For a fixed constant θ, consider the random variables

$$U = X\cos\theta - Y\sin\theta, \quad V = X\sin\theta + Y\cos\theta.$$

(b) Why are U and V jointly Gaussian? 3p

(c) What is the distribution of $U^2 + V^2$? 3p

Problem 3. The number X of electrons counted by a receiver in an optical communication system is a Poisson random variable with rate λ, when a signal is present, and with rate $\lambda_0 < \lambda$, when a signal is absent. Suppose that a signal is present with probability p.

(a) Compute the expected value of the electrons counted. 3p

(b) Suppose the receiver uses the following decision rule:

If $X > t$, decide signal present, otherwise, decide signal absent.

Find the error probability of the above decision rule. 3p

Problem 4. A WSS random process with a PSD $S_X(f) = \frac{1}{1+f^2}$ is input to a filter. The filter is to be designed so that the output process is white, i.t., has a constant PSD. Find the transfer function of the whitening filter for this input process. Be sure that the filter is causal. 4p

Problem 5. Let $Y_n = X_n + \beta X_{n-1}$, where $\{X_n\}$ is a zero-mean, first-order autoregressive process with autocorrelation

$$R_X(k) = \sigma^2\alpha^{|k|}, \quad |\alpha| < 1.$$

Compute the power spectral density functions of X and Y. 4p

Problem 6. The spectrum of a stationary stochastic process is to be estimated from the following data:

$$x[n] = \{0.6,\ -0.7,\ 0.2,\ 0.3\}.$$

Due to the small sample support, a simple AR(1)-model is exploited:

$$x[n] + a_1 x[n-1] = e[n].$$

Determine estimates of the AR-parameter a_1 and the white noise variance σ_e^2. Based on these, give a parametric estimate of the spectrum $P_X(e^{j\omega})$. 4p