Written test for the examination
"Random Processes with Applications", 2009-10-22, 14:00 - 18:00
On duty: Rossitza Dodunekova 772 3534.
Allowed material: The handbook *Beta*, *Collection of formulas for MVE135*, calculators approved by Chalmers.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. Suppose Z_1, Z_2, and Z_3 are independent Gaussian random variables with expectation zero and variance one. What is the distribution of $Z_4 = 3/5\,Z_1 + 4/5\,Z_2$? Write the joint probability density function of Z_1 and Z_4. Show that $P\{Z_1 < Z_3\} = 1/2$. 6p

Problem 2. The input to a communication channel is a random variable X with equiprobable values -1 and 1. The output of the channel is the random variable $Y = X + N$, where the noise random variable N is independent of X and has Gaussian distribution with mean zero and variance one. Find the probability density function of the output. Suppose you observe a negative output and have to decide whether the input was 1 or -1. What would your choice be? 6p

Problem 3. Messages arrive in a multiplexer according to a Poisson process of rate λ messages per second.

(a) Suppose it is known that exactly one message has arrived in the time interval $[0, t_0]$. Find the probability density function of the arrival time. (3)

(b) Assume $\lambda = 10$. Use the Central Limit Theorem to estimate the probability for more then 640 messages in one minute. (3)

Problem 4. Let $Y_n = X_n + \beta X_{n-1}$, where $\{X_n\}$ is a zero-mean, first-order autoregressive process with autocorrelation

$$R_X(k) = \sigma^2 \alpha^{|k|}, \quad |\alpha| < 1.$$

Compute the power spectral density of Y. Find a value of β for which $\{Y_n\}$ is the white noise process. 6p

Problem 5. Y_n is a wide sense stationary process defined as

$$Y_n = \frac{1}{2} Y_{n-1} + X_n,$$

where X_n is the white-noise process of average power 1. Compute the autocorrelation function of Y_n. Find the unite impulse response of the filter producing the best linear predictor of Y_n from Y_{n-2} and Y_{n-3} and compute the mean-square estimation error. 6p