Solutions: Chapter 7 Problems 1, 12, 15, 29, 42, 47, 49

Problem 1. I consider sampling WITH replacement. For an answer in the case of sampling without replacement consult the book page A34.

Population distribution

Values	1	2	4	8
Probab.	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Population mean and variance: $\mu = 3.4$, $\sigma^2 = 6.24$. The list of $\bar{X}$ values and their frequencies for $n = 2$ observations taken with replacement:

	1	2	4	8	Total prob.
1	1.0 (1/25)	1.5 (2/25)	2.5 (1/25)	4.5 (1/25)	1/5
2	1.5 (2/25)	2.0 (4/25)	3.0 (2/25)	5.0 (2/25)	2/5
4	2.5 (1/25)	3.0 (2/25)	4.0 (1/25)	6.0 (1/25)	1/5
8	4.5 (1/25)	5.0 (2/25)	6.0 (1/25)	8.0 (1/25)	1/5
Tot. prob.	1/5	2/5	1/5	1/5	1

The sampling distribution of $\bar{X}$:

Values	1	1.5	2	2.5	3	4	4.5	5	6	8
Probab.	$\frac{1}{25}$	$\frac{4}{25}$	$\frac{4}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{1}{25}$	$\frac{2}{25}$	$\frac{4}{25}$	$\frac{2}{25}$	$\frac{1}{25}$

$\mathrm{E}(\bar{X}) = 3.4 = \mu$, $\mathrm{E}(\bar{X}^2) = 14.68$, $\mathrm{Var}(\bar{X}) = 3.12 = \frac{\sigma^2}{n}$.

Problem 15. Normal approximation: $\frac{\bar{X}-\mu}{s_{\bar{X}}} \in \mathrm{N}(0,1)$. Approximate 95% one-sided CI for μ:

$0.95 = \mathrm{P}(\frac{\bar{X}-\mu}{s_{\bar{X}}} < 1.645) = \mathrm{P}(\bar{X} - 1.645 s_{\bar{X}} < \mu < \infty)$.

Problem 29. Population total $\tau = N\mu$, its unbiased estimate $T = N\bar{X}$:

$\mathrm{Var}(T) = N^2 \frac{\sigma^2}{n}$ for an IID sample,

$\mathrm{Var}(T) = N^2 \frac{\sigma^2}{n}(1 - \frac{n-1}{N-1})$ for a simple random sample.

Problem 42. Compute from the data

$\bar{X} = 3.2$, $\bar{Y} = 100$, $s_x^2 = 2.28$, $s_x = 1.51$ $s_y^2 = 1010.1$, $s_y = 31.78$, $s_{xy} = 40.40$, $\hat{\rho} = 0.84$

a) $R = 31.25$ dollars per week per person

b) Since $s_R^2 = \frac{1}{n\bar{X}^2}(R^2 s_x^2 + s_y^2 - 2Rs_{xy}) = 1.22$ an approximate CI is $31.25 \pm 1.96 \cdot 1.10 = 31.25 \pm 2.17$

c) $T = N\bar{Y} = 10,000,000$, $s_T^2 = 1010.1 \cdot 10^8$, a 90% approximate CI for τ is $10,000,000 \pm 1.645 \cdot 317800 = 10,000,000 \pm 522,781$

Problem 47. a) $N = 2010$, $L = 7$, $\bar{\sigma} = 17.04$, $n = 100$

μ_l	5.4	16.3	24.3	34.5	42.1	50.1	63.8
W_l	.196	.229	.196	.166	.084	.056	.074
σ_l	8.3	13.3	15.1	19.8	24.5	26.0	35.2
Optimal $n\frac{W_l\sigma_l}{\bar{\sigma}_l}$	9.6	17.9	17.4	19.3	12.1	8.5	15.3
Proportional nW_l	19.6	22.9	19.6	16.6	8.4	5.6	7.4

b) $\mathrm{Var}(\bar{X}_{so}) = \frac{\bar{\sigma}^2}{n} = 2.90$, $\mathrm{Var}(\bar{X}_{sp}) = \frac{\overline{\sigma^2}}{n} = 3.44$, $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n} = 6.21$.

c) $\mu = 26.49$, $\sigma^2 = 621$.

d) If $n_1 = \ldots = n_7 = 10$ and $n = 70$, then $\mathrm{Var}(\bar{X}_s) = 4.52$. Find sample size x such that $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{x} = 4.52$: $x = 137.4$.

e) If $n = 70$, then $\mathrm{Var}(\bar{X}_{sp}) = 4.91$. Find sample size x such that $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{x} = 4.91$: $x = 126.4$.

Problem 49. Stratified population

$N = 5$, $L = 2$, $W_1 = 0.6$, $W_2 = 0.4$, $\mu_1 = 1.67$, $\mu_2 = 6$, $\sigma_1^2 = 0.21$, $\sigma_2^2 = 4$. Given $n_1 = n_2 = 1$ and $n = 2$, the sampling distribution of $\bar{X}_s = 0.6X_1 + 0.4X_2$:

	1	2	Total prob.
4	2.2 (1/6)	2.8 (2/6)	1/2
8	3.8 (1/6)	4.4 (2/6)	1/2
Tot. prob.	1/6	2/3	1

$\mathrm{E}(\bar{X}_s) = 3.4 = \mu$, $\mathrm{E}(\bar{X}_s)^2 = 12.28$, $\mathrm{Var}(\bar{X}_s) = 0.72 = 0.36\sigma_1^2 + 0.16\sigma_2^2$.