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Solutions: Chapter 11

Problems 1, 11, 15, 19, 34, 35, 46

Problem 1.

a)
$$\bar{X} = 0.5546$$
, $\bar{Y} = 1.6240$, $\bar{Y} - \bar{X} = 1.0694$

b)
$$s_x^2 = 0.2163$$
, $s_y^2 = 1.1795$, $s_p^2 = 0.7667$

c)
$$s_{\bar{y}-\bar{x}} = 0.5874$$

d) exact 90% CI for
$$(\mu_y - \mu_x)$$
 is 1.0694 ± 1.1128

- e) two-sided
- f) observed T = 1.8206, df = 7, two-sided P = 0.1115 [=2*tcdf(-1.8206,7)]
- g) no

h) b:
$$\sigma^2=1$$
, c: $s_{\bar{y}-\bar{x}}=0.0.6708$, d: 1.0694 ± 1.1035 , f: $Z=1.5942$ two-sided $P=0.11$, g: no

Problem 15.

Approximate 95% CI for $(\mu_x - \mu_y)$ is $\bar{X} - \bar{Y} \pm 1.96 \cdot 10 \cdot \sqrt{\frac{2}{n}}$. CI width $2 = 55.44/\sqrt{n}$. Thus $n \approx 768$.

Problem 19.

a) $\bar{X}=10.693, \bar{Y}=6.750, s_x^2=23.226, s_y^2=12.978, s_{\bar{x}-\bar{y}}=\sqrt{s_{\bar{x}}^2+s_{\bar{y}}^2}=1.903,$ observed T=2.072, df = 18, two-sided P=0.053 [=2*tcdf(-2.072,18)] assuming equal variances

b) Ranks in the pooled sample

$$X: 1, 8, 9, 11, 13, 14, 17, 18, 19, 20$$

 $Y: 2, 2, 4, 5, 6, 7, 10, 12, 15, 16$

Wilcoxon rank sum test $R_x = 130$, $R_y = 80$, two-sided 0.05 < P < 0.10 [see page A22].

- c) Nonparametric since both normplot(x) and normplot(y) show non-normality of the data distribution
- d) Order in the pooled sample x-yyyyyy-xx-y-xy-xx-yy-xxxx $P(X < Y) \approx \frac{\text{number of } (x_i < y_j)}{\text{total number of pairs } (x_i, y_j)} = \frac{10 + 4 + 4 + 3 + 2 + 2}{100} = 0.25 \text{ implies } \hat{\pi} = 0.75$
- e) The matlab commands u=x(random('unid',10,10,1000));

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\begin{array}{l} v \! = \! y(random('unid',10,10,1000)); \\ N \! = \! zeros(1,1000); \\ for k \! = \! 1:1000 \ for i \! = \! 1:10 \ for j \! = \! 1:10 \\ N(k) \! = \! N(k) \! + \! (u(i,k) \! > \! v(j,k)); \\ end,end,end \\ P \! = \! N/100; \\ hist(P,20) \\ std(P) \end{array}
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estimate the sampling distribution of $\hat{\pi}$ with $s_{\hat{\pi}} = 0.1187$.

f) Matlab commands

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c1=prctile(P,5)
c2=prctile(P,95)
give a 90% CI for \pi: (2\hat{\pi} - c2; 2\hat{\pi} - c1) = (0.58; 0.96).
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Problem 34.

$$ar{X} = 85.26, \, s_x = 21.20, \, s_{\bar{x}} = 5.47, \, n = 15$$

 $ar{Y} = 84.82, \, s_y = 21.55, \, s_{\bar{y}} = 5.57, \, m = 15$
 $ar{D} = ar{X} - ar{Y} = 0.44$

Paired samples $s_d = 4.63, s_{\bar{d}} = 1.20$ Independent samples $s_{\bar{x}-\bar{y}} = 7.81$

To test $H_0: \mu_x=\mu_y$ against $H_1: \mu_x\neq \mu_y$ assume $D\in \mathcal{N}(\mu,\sigma^2)$ and apply one-sample t-test

$$T = \frac{\bar{D}}{s_d} = 0.368$$
, df = 14, two-sided $P = 0.718$, can not reject H_0 .

Without normality assumption apply the Wilcoxon signed rank test. Matlab command $\operatorname{signrank}(x,y)$

computes the two-sided P = 0.604. We can not reject H_0 .

Problem 46.

- a) room with a window \leftarrow rich patient \rightarrow recovers faster
- b) smoker \leftarrow loser \rightarrow wife gets cancer
- c) no breakfast \leftarrow lack of discipline \rightarrow accident
- d) choose to change the school \leftarrow lower grades before \rightarrow lower grades after
- e), i)?
- f) abstain from alcohol \leftarrow poor health
- g) marijuana \leftarrow schizophrenia
- h) total time together = time before wedding + time after wedding