Solutions: Chapter 9 Problems 1, 3, 7, 14, 20, 27, 33

Problem 3.
b) $\mathrm{Pw}(p) \approx \mathrm{P}(Z < \frac{40-100p}{10\sqrt{pq}}) + \mathrm{P}(Z > \frac{60-100p}{10\sqrt{pq}})$:

p	.3	.35	.4	.45	.5	.55	.6	.65	.7
$\mathrm{Pw}(p)$	.986	.853	.500	.159	.046	.159	.500	.853	.986

Problem 7. Likelihood function $L(\lambda) = e^{-\lambda n}\lambda^{x_1+\ldots+x_n}\prod_{i=1}^{n}\frac{1}{x_i!}$
Reject H_0 for small
$\frac{L(\lambda_0)}{L(\lambda_1)} = e^{-n(\lambda_0-\lambda_1)}(\frac{\lambda_0}{\lambda_1})^{x_1+\ldots+x_n}$ or for large $y = x_1 + \ldots + x_n$.
Test statistic $Y = X_1 + \ldots + X_n$ has null distribution $\mathrm{Pois}(n\lambda_0)$.

Problem 14.
Exact 95% CI for σ^2 if $n = 15$: $(0.536s^2; 2.487s^2)$. Reject $H_0 : \sigma = 1$ if $s^2 > 1.866$ or $s^2 < 0.402$.

Problem 20.
a) Two-sided P-value = 0.134.

b) One-sided P-value = 0.067.

Problem 27.
Multinomial model $(X_1, X_2, X_3) \in \mathrm{Mn}(190, p_1, p_2, p_3)$. Composite null hypothesis (Hardy-Weinberg Equilibrium)
$H_0 : p_1 = (1-\theta)^2, p_2 = 2\theta(1-\theta), p_3 = \theta^2$.
Likelihood function and MLE
$L(\theta) = \binom{190}{10,68,112}2^{68}\theta^{292}(1-\theta)^{88}$, $\hat{\theta} = \frac{88}{380} = 0.768$.
Pearson's chi-square test:

cell	1	2	3	Total
observed	10	68	112	190
expected	10.23	67.71	112.07	190

Observed $X^2 = 0.0065$, df = 1, P-value $= 2(1 - \Phi(\sqrt{0.0065})) = 0.94$.

Problem 33.
a) Number of heads $Y \in \mathrm{Bin}(17950; p)$. For $H_0 : p = 0.5$ the observed $Z = 3.46$. Reject H_0.

b) Pearson's chi-square test for the simple null hypothesis
$H_0 : p_0 = (0.5)^5 = 0.031,\ p_1 = 5\cdot(0.5)^5 = 0.156,\ p_2 = 10\cdot(0.5)^5 = 0.313,$
$p_3 = 10\cdot(0.5)^5 = 0.313,\ p_4 = 5\cdot(0.5)^5 = 0.156,\ p_5 = (0.5)^5 = 0.031$

number of heads	0	1	2	3	4	5	Total
observed	100	524	1080	1126	655	105	3590
expected	112.2	560.9	1121.9	1121.9	560.9	112.2	3590

Observed $X^2 = 21.58$, df $= 5$, P-value $= 0.001$.

c) Composite null hypothesis

$$H_0 : p_i = \binom{5}{i} p^i (1-p)^{5-i}, \ i = 0, 1, 2, 3, 4, 5.$$

Pearson's chi-square test based on the MLE $\hat{p} = 0.5129$

number of heads	0	1	2	3	4	5	Total
observed	100	524	1080	1126	655	105	3590
expected	98.4	518.3	1091.5	1149.3	605.1	127.4	3590

Observed $X^2 = 8.74$, df $= 4$, P-value $= 0.07$. Accept H_0 at 5% level.