TENTAMEN I TMA 321, MS, F/KF 2014-01-17, CIRKUS
LÖSNINGSFÖRSLAG, OLLE NERMAN

1. a. $M_X(t) = E[e^{tX}] = (1-p) + pe^t = 0{,}3 + 0{,}7e^t$

b. $M_Y(t) = E[e^{(3-4X)t}] = e^3 \, E[e^{(-4t)X}] = e^3 M_X(-4t) = e^3(0{,}3 + 0{,}7e^{-4t})$

c. $E[Y] = M_Y'(0) = (3 - 4\,E[X]) = 3 - 4 \cdot 0{,}7 = 0{,}2$

$VAR[Y] = M_Y''(0) - (M'(0))^2 = VAR\,[3-4X] = (-4)^2\, VAR[X] = 16 \cdot 0{,}7 \cdot 0{,}3 =$
$= 3{,}36$

2. a. $P(X>0) = 1 - \Phi\left(\frac{0-4}{\sqrt{9}}\right) = 1 - \Phi\left(\frac{4}{3}\right) \approx 1 - 0{,}9082 \approx 0{,}0918$

b. $P(|X-4|>3) = \Phi\left(\frac{1-4}{\sqrt{9}}\right) + \left(1 - \Phi\left(\frac{7-4}{\sqrt{9}}\right)\right) = 2 \cdot (1 - \Phi(1)) =$
$= 2 \cdot (1 - 0{,}8413) = 2 \cdot 0{,}1587 \approx 0{,}3174$

c. $P(|X|>5) = P(X \le -5) + P(X>5) = \Phi\left(\frac{-5-4}{3}\right) + \left(1 - \Phi\left(\frac{5-4}{3}\right)\right) =$
$= 2 - \Phi(3) - \Phi\left(\frac{1}{3}\right) \approx 2 - 0{,}9987 - 0{,}6293 =$
$= 0{,}372$

3. $N(20) \sim$ POISSONFÖRDELAD $(20 \cdot c)$

$L(c) = \frac{(20c)^k e^{-20c}}{k!}$ om $N(20) = k$

$\frac{d \ln L_c}{dc} = 0 \Rightarrow c = \frac{k}{20}$ $\therefore$ $\hat{c} = \frac{N(20)}{20} = \frac{154}{20} = 7{,}7$

$N(20) \approx$ NORMALFÖRDELAD $(20c, 20c)$. ANVÄND

$\frac{N(20) - 20c}{\sqrt{20c}} \approx \frac{N(20) - 20c}{\sqrt{N(20)}} \approx N(0,1) \Longrightarrow$

$c = \hat{c} \pm \frac{1{,}96 \cdot \sqrt{\hat{c}}}{\sqrt{20}} = 7{,}7 \pm \frac{1{,}96 \cdot \sqrt{7{,}7}}{\sqrt{20}}$ $(\approx 95\%)$

4 a. $F_Y(y) = P(Y \le y) = \begin{cases} 0 & \text{om } y < 0 \\ \left(\frac{y}{10}\right)^3 & \text{om } y \in [0,10] \\ 1 & \text{om } y > 10 \end{cases}$ | $f_Y(y) = F_Y'(y) = \frac{3y^2}{10^3}$ om $y \in [0,10]$ (0 f.ö.)

b. $E[Y] = \int_0^{10} y \cdot \frac{3y^2}{10^3}\, dy = \left[\frac{3\, y^4}{4 \cdot 10^3}\right]_0^{10} = \frac{30}{4} = 7{,}5$

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5\. a. ML-SKATTNING (=MOMENTSKATTNING) AV $\lambda = \frac{1}{\bar{X}}$, DÄR $\bar{X} = \frac{1}{15}\left(\sum_{i=1}^{15} x_i\right)$ $\therefore$ $\hat{\lambda} = \frac{1}{2,7}$

b. $E[X_i] = \frac{1}{\lambda}$ $\therefore$ $\widehat{E[X_i]} = 2,7$

c. $\text{MED}(X_i) = m \Rightarrow 1 - e^{-\lambda m} = 0,5 \Rightarrow m = \frac{\ln 2}{\lambda}$

$\hat{m} = \ln 2 / \hat{\lambda} = \ln(2)\cdot 2,7$

d. $P(X_i < 2) = P(X_i \le 2) = 1 - e^{-\lambda\cdot 2}$ $\quad \widehat{P(X_i<2)} = 1 - e^{-\hat{\lambda}\cdot 2} = 1 - e^{-\frac{2}{2,7}}$

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6\. a. $t$-INTERVALL BASERAT PÅ 14 FRIHETSGRADER

$F_{t(14)}(2,977) = 0,995$ GER KONF.INTERVALL

$\mu = \hat{\mu} \pm \frac{2,977\, s}{\sqrt{15}} = 10,25 \pm \frac{2,977\cdot 0,37}{\sqrt{15}}$ (99%)

b. $t$-TEST, ENSIDIGT, OBSERVERAD STATISTIKA

$t = \sqrt{15}\,\frac{10,25 - 10}{0,37} \geq 1,761$ $\quad (F_{t(14)}(1,761) = 0,95)$

$\Rightarrow$ $H_0$ FÖRKASTAD PÅ SIGNIFIKANSNIVÅN 5%

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7\. a. PUNKTSKATTNING AV $\alpha + \beta\cdot 2$. $\hat{\alpha} + \hat{\beta}\,2$

$\hat{\beta} = 0,32$ $\quad \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = \frac{10,3}{10} - 0,32\cdot 0,42$

b. $\hat{\beta} \sim \text{NORMALF.}\left(\beta, \sigma^2 \frac{1}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right) = N\left(\beta, 4\cdot\frac{1}{14\cdot 6,3}\right)$

c. $\beta = \hat{\beta} \pm 1,96\cdot\sqrt{4\cdot\frac{1}{14\cdot 6,3}}$ (95%)

(NORMALFÖRDELNINGSBASERAT!)

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8\. a. $P(A\cap B\cap C) = 1 - P(\complement A \cup \complement B \cup \complement C) \geq$

$\geq 1 - (P(\complement A) + P(\complement B) + P(\complement C)) \geq 1 - 0,3 - 0,3 - 0,3 = 0,1$

b. $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{0,7}$ $\quad$ MEN $P(A\cup B) \le 1$

$\Rightarrow P(A\cup B) = P(A) + P(B) - P(A\cap B) = 1,4 - P(A\cap B) \le 1$

$\therefore$ $P(A\cap B) \geq 0,4$ VILKET GER OLIKHETEN.