CHALMERS | UNIVERSITY OF GOTHENBURG

A. Lang & A. Petersson

Financial Time Series TMS088/MSA410 – LP4 2017/18

Project 1

The file *exchangerate.mat* contains monthly observations of the Australian Trade Weighted Index, a weighted sum of exchange rates between the Australian dollar and other currencies[1], in the form of the time series

$$X = (X_t, t = 1, \ldots, 205) \tag{1}$$

with t representing number of months after Jan 1978. You are to work with this data set using MATLAB. To import the contents of the mat-file, make sure your current folder in MATLAB contains the file and write

```
data = importdata('exchangerate.mat');
```

1. (3 points) Create the time series $Y = (Y_t, t = 1, \ldots, 204)$ of *absolute returns*, given by

$$Y_t = X_{t+1} - X_t.$$

   Also create the time series $Z = (Z_t, t = 1, \ldots, 204)$ of *log returns*, given by

$$Z_t = \log(X_{t+1}) - \log(X_t).$$

   After this, mean correct all three series, i.e., subtract the sample mean from each series. Plot them. Which, if any, of them do you think can be modeled as a stationary time series? Motivate your answer. *After this, you may assume that all three time series have mean zero.*

2. (5 points) Suppose that someone claims that all three time series are iid (independent and identically distributed). Implement (i.e., write your own code that calculates the statistic, do not use the built-in MATLAB function) the Ljung-Box test with $h = 20$ and $\alpha = 0.05$ to investigate this claim. What is the result for each time series? Report all three p-values and whether you accept/reject the claim. Also plot the ACF for each time series.

---

[1]Source: https://datamarket.com/

Please turn!

3. (10 points) Let's say you wish to predict the values of the log returns, Z, at one month given the values of the previous 20 months. To do this, first split the time series into a *training data set*, consisting of the first 102 observations, and a *test data set*, consisting of the rest of the time series Z. Next, compute the sample ACVF $\hat{\gamma}$ *using the data from the training data set only*. Then compute all linear forecasts

$$b_n^l(z_{n-1}, \ldots, z_{n-20}) = a_1 z_{n-1} + a_2 z_{n-2} + \ldots + a_{20} z_{n-20}$$

for $n = 103, 104, \ldots, 204$ by solving the equations in Proposition 2.3.5, replacing the exact ACVF γ with the sample ACVF you computed from the training data set. Evaluate the performance of your predictions by plotting them in the same figure as the test data and by computing the error

$$\frac{1}{102} \sum_{n=103}^{204} \left(b_n^l(z_{n-1}, \ldots, z_{n-20}) - z_n \right)^2.$$

Compare this to the error you get from the naive 'prediction' of just using the mean (which is zero since you have a mean corrected series), i.e., compute the error

$$\frac{1}{102} \sum_{n=103}^{204} (\hat{\mu} - z_n)^2 = \frac{1}{102} \sum_{n=103}^{204} z_n^2.$$

Discuss your results.

4. (3 points) It is sometimes claimed that log returns can be modeled as a Gaussian time series. Investigate this by either implementing your favorite statistical test for normality of Z or by creating a qq-plot (if you do the latter, simulate some Gaussian data for a comparison qq-plot). Also repeat Exercise 2., but instead of testing all time series, now test only $|Z|$, the absolute value of the log returns (remember to mean correct it first). Discuss your results.

5. (9 points) Assume that the stochastic process $X = (X_t, t \in \mathbb{Z})$ is given by

$$X_t := m_t + Y_t,$$

where Y is stationary with $\mathbb{E}[Y_t] = 0$ for all $t \in \mathbb{Z}$ and that the trend is given by the polynomial

$$m_t := \sum_{j=0}^{q} a_j t^j$$

for some coefficients $a_j \in \mathbb{R}$, $j = 0, \ldots, q$.

a) Show that $\nabla^q m_t = q!\, a_q$. (*Hint: Start with $q = 1$ and ∇m_t.*)

b) Show that $\nabla^q Y$ is stationary.

c) Conclude that $\nabla^q X$ is a mean $q!\, a_q$, stationary process.

Deadline: April 23, 2018.
Requirement: You must do this project in MATLAB. For this project there are 30 points available. To qualify for bonus points you need to score at least 10 points. After that, every 2.5 points you score on this project will translate into 0.5 bonus points on the exam.
Formalities: You are strongly encouraged to work in pairs. Send your project report as one pdf document to both

annika.lang@chalmers.se and *annika.lang.chalmers@analys.urkund.se.*

Your report should include all plots, explanations, and answers to the questions as well as your implemented code in an appendix. You should try to write your report in LaTeX. If you do not, it is acceptable to scan your handwritten solutions to the theoretical parts of the project and include them in the pdf file. Put the original in the folder in front of *MVL2086* no later than *April 24, 10.00*. The code should include comments to be readable.