Tentamentsskrivning i Statistisk slutledning, TM, 5p.

Tid: onsdagen den 28 maj 2003 kl 14.15-18.15.
Examinator och jour: Serik Sagitov, tel. 772-5351, rum MC 1421.
Hjälpmedel: miniräknare, egen formelsamling (4 sidor på 2 blad A4) samt utdelade tabeller.
För "G" fordras 12 poäng, för "VG" - 17 poäng, för "MVG" - 22 poäng.

Important: carrying out a test make sure

- to state the hypotheses tested,
- state the statistical test you choose,
- explain your choice of the test by referring to the conditions assumed by the test.

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1. (5 marks) Consider the following three point estimates:

MLE = maximum likelihood estimate,
MAP = maximum aposteriori estimate,
PME = posterior mean estimate.

a. Each of the three estimates is optimal in a certain setting. Describe in brief these three different settings.

b. In what sense the MLE can be viewed as a particular case of MAP?

c. Suppose the posterior distribution is positively skewed. Which one of the following relationships holds: MAP $<$ PME, MAP $=$ PME, MAP $>$ PME? Draw a graph illustrating your answer.

2. (6 marks) A series of 33 insect traps was set out across sand dunes and the numbers of insects from a certain taxonomic group (Staphylinoidea) were counted.

Individuals in a trap	0	1	2	3	4	5	6	Total
Numbers of traps	10	9	5	5	1	2	1	33

a. Under what circumstances the Poisson distibution Pois(λ) (with pms $p_k = \frac{\lambda^k}{k!}e^{-\lambda}$ so that $p_k = \frac{\lambda}{k}p_{k-1}$) might be an appropriate model for the distribution of insects? Find the MLE of the parameter λ.

b. State the corresponding null hypothesis in terms of seven probabilities for X, the number of insects in a trap:

$$\mathrm{P}(X = 0), \ldots, \mathrm{P}(X = 5), \mathrm{P}(X \geq 6).$$

c. Test this hypothesis using the data.

d. How and why the small cells should be grouped together when applying a chi-square test?

3. (7 marks) Why do older people often seem not to remember things as well as younger people? Do they not pay attention? Do they just not process the material as thoroughly? One theory regarding memory is that verbal material is remembered as a function of the degree to which is was processed when it was initially presented.

Eysenck (1974) randomly assigned 50 younger subjects and 50 older (between 55 and 65 years old) to one of five learning groups. The Counting group was asked to read through a list of words and count the number of letters in each word. This involved the lowest level of processing. The Rhyming group was asked to read each word and think of a word that rhymed with it. The Adjective group was asked to give an adjective that could reasonably be used to modify each word in the list. The Imagery group was instructed to form vivid images of each word, and this was assumed to require the deepest level of processing.

None of these four groups was told they would later be asked to recall the items. Finally, the Intentional group was asked to memorize the words for later recall. After the subjects had gone through the list of 27 items three times they were asked to write down all the words they could remember. The table presents the numbers of words recalled correctly by each of 100 participants.

	Younger	Older
Counting	8, 6, 4, 6, 7, 6, 5, 7, 9, 7	9, 8, 6, 8, 10, 4, 6, 5, 7, 7
Rhyming	10, 7, 8, 10, 4, 7, 10, 6, 7, 7	7, 9, 6, 6, 6, 11, 6, 3, 8, 7
Adjective	14, 11, 18, 14, 13, 22, 17, 16, 12, 11	11, 13, 8, 6, 14, 11, 13, 13, 10, 11
Imagery	20, 16, 16, 15, 18, 16, 20, 22, 14, 19	12, 11, 16, 11, 9, 23, 12, 10, 19, 11
Intentional	21, 19, 17, 15, 22, 16, 22, 22, 18, 21	10, 19, 14, 5. 10, 11, 14, 15, 11, 11

The next table summarizes the means and variances of interest.

	Younger	Older	Total
Counting	mean = 6.5, var = 2.06	mean = 7.0, var = 3.33	mean = 6.75, var = 2.62
Rhyming	mean = 7.6, var = 3.82	mean = 6.9, var = 4.54	mean = 7.25, var = 4.09
Adjective	mean = 14.8, var = 12.18	mean = 11.0, var = 6.22	mean = 12.90, var = 12.52
Imagery	mean = 17.6, var = 6.71	mean = 13.4, var = 20.27	mean = 15.50, var = 17.42
Intentional	mean = 19.3, var = 7.12	mean = 12.0, var = 14.00	mean = 15.65, var = 24.03
Total	mean = 13.16, var = 33.48	mean = 10.06, var = 16.06	mean = 11.61, var = 26.95

a. Put together an ANOVA table for the data. Show clearly how calculations of the sums of squares are performed using the second table only. (Hint: calculate the interaction sum of squares from the other sums of squares.)

b. Draw graphs comparing mean performance of older and younger subjects for different levels of word processing. What do the graphs say about the interaction between the two factors? Is the interaction statistically significant?

c. Formulate and test at 1% level a null hypothesis concerning the age factor of the study.

d. Does the study confirm that words are remembered better if processed properly?

e. What assumptions do you make for your ANOVA analysis? How consistent are these assumptions with the variation observed in the data?

4. (6 points) Suppose a population consists of 3 strata with unknown strata means μ_1, μ_2, μ_3, and different standard deviations σ_1, σ_2, σ_3. Assume that (large) strata sizes are in proportion $1 : 2 : 3$ while the strata standard deviations relate as $\sigma_1 : \sigma_2 : \sigma_3 = 3 : 2 : 1$.

a. Express the total population mean μ in terms of parameters μ_1, μ_2, μ_3. Show that a stratified sample mean with sample size allocation $n = n_1 + n_2 + n_3$ is an unbiased estimate of μ.

b. Describe the random, proportional, and optimal allocation schemes for $n = 100$. Compare the standard errors of the correspondent estimates of μ. In general it is often difficult to apply an optimal allocation scheme - why?

c. Suppose additionally that $\mu_1 = \mu_2 = \mu_3 = \mu$. What in this case is the optimal allocation scheme for $n = 100$ observations if you want to estimate the total population mean μ? Compare the standard error of this estimate with those obtained in b).

5. (6 marks) In a raid on a coffee shop, Bayesian trading inspectors take a random sample of 20 packets of coffee, each of nominal weight 125 g. The data they obtain are (weights in grams):

105.3, 113.3, 114.5, 121.2, 122.9, 123.7, 124.0, 124.6, 124.9, 124.9,
124.9, 125.1, 125.5, 125.9, 126.8, 127.7, 128.2, 128.3, 128.5, 130.2,

($\sum x_i = 2470.4$, $\sum x_i^2 = 305829.0$). They model these data as independent values from a Normal $\mathrm{N}(\mu, \sigma^2)$ distribution with known $\sigma^2 = 4$. For μ they assume a prior distribution $\mathrm{N}(\mu_0, \sigma_0^2)$ with $\mu_0 = 126$ and $\sigma_0^2 = 1$. The inspectors can impose a fine if their 95% credibility interval falls wholly below the claimed value of $\mu = 125$ g.

a. Find the inspector's posterior distribution. Express the mean of this distibution as a weighted average of μ_0 and the sample mean.

b. Show that the inspector's 95% credibility interval falls wholly below 125 g; they therefore impose a fine on the owners of the coffee shop.

c. Comment briefly as to whether the inspectors are justified in imposing a fine on the basis of this sample.

Statistical tables supplied:
1. Normal distribution table
2. Chi-square distribution table
3. t-distribution table
4. F-distribution table

Partial answers and solutions are also welcome. Good luck!

ANSWERS

1c. MAP < PME

2a. If the insects are spread randomly across the dunes.

2b. MLE $\hat{\lambda}$= 54/33=1.636. Expected counts: 6.4, 10.5, 8.6, 4.7, 1.9, 0.6, 0.2
Combined counts:
observed 10, 9, 5, 9
expected 6.4, 10.5, 8.6, 7.5
$X^2 = 4.05$, df = 2. Can not reject H_0.

3a. The ANOVA table

Source	SS	df	MS	F	P
Age	240.3	1	240.3	29.94	< 0.01
Process	1515	4	378.7	47.19	< 0.01
Interaction	190.3	4	47.58	5.93	< 0.01
Error	722.3	90	8.026		
Total	2668	99			

3b.

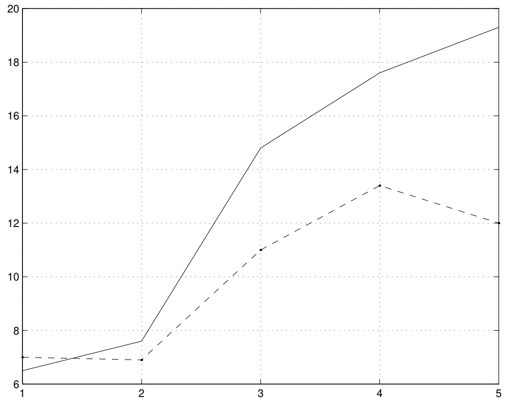

Figure 1: *Younger vs older performance*

4b. Assuming $\sigma_1 = 3x$, $\sigma_2 = 2x$, $\sigma_3 = x$

	n_1	n_2	n_3	$\sigma_{\bar{X}}$
Random	X_1	X_2	X_3	$0.182x + c$
Proportional	17	33	50	$0.182x$
Optimal	30	40	30	$0.167x$

where $(X_1, X_2, X_3) \in Mn(100, \frac{1}{6}, \frac{2}{6}, \frac{3}{6})$

4c. $n_1 = n_2 = 0$, $n_3 = 100$, $\sigma_{\bar{X}} = 0.1x < 0.167x$

5a. $\bar{X} = 123.52$, $\mu_1 = \frac{126+617.6}{6} = 123.93$, $\sigma_1^2 = 0.167$.

5b. (123.13, 124.73).

5c. $s = 6.01$ contradicts $\sigma = 2$ assumption. Three outliers (105.3, 113.3, 114.5) contradict the normality assumption.