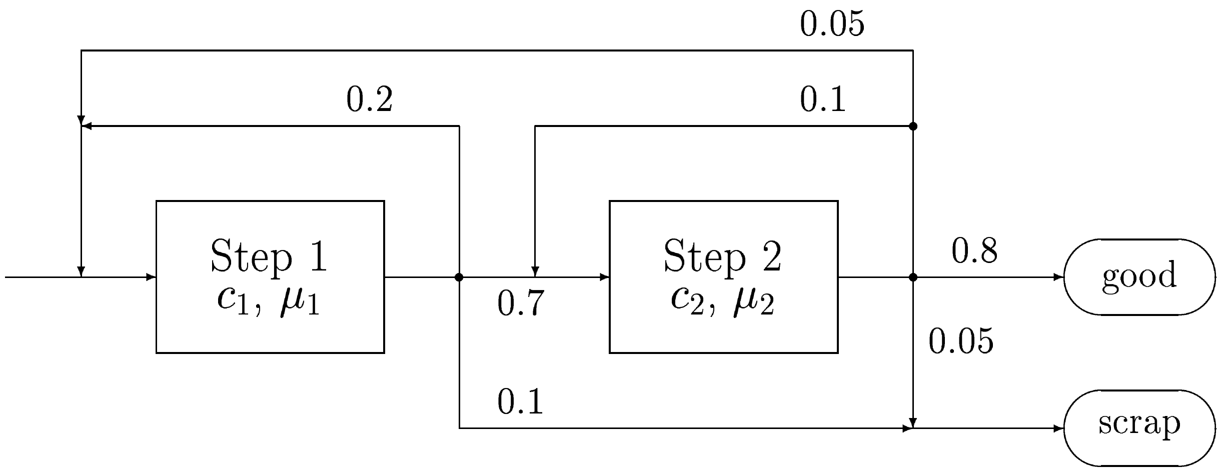
Figur 2.7, sida 52



Tillverkningsprocessen i exempel 2.8

Transitionssannolikheterna

Transition	Sannolikhet
Step 1 → Step 1	$P(1,1) = 0.2$
Step 1 → Step 2	$P(1,2) = 0.7$
Step 1 → scrap	$P(1,s) = 0.1$
Step 2 → Step 1	$P(2,1) = 0.05$
Step 2 → Step 2	$P(2,2) = 0.1$
Step 2 → scrap	$P(2,s) = 0.05$
Step 2 → good	$P(2,g) = 0.8$

sammanfattas i en transitionsmatris

$$\mathbf{P} = \begin{bmatrix} 0.2 & 0.7 & 0.1 & 0.0 \\ 0.05 & 0.1 & 0.05 & 0.8 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

över tillståndsrummet

$$E = \{1, 2, s, g\} = \{1, 2\} \cup \{s\} \cup \{g\}$$

Av intresse i sammanhanget är

$$F(1, s) = \Pr\{\text{detaljen kasseras}\}$$

och

$$F(1, g) = 1 - F(1, s) = \Pr\{\text{detaljen godkännes}\}$$

samt genomsnittlig tillverkningskostnad

Exempel 2.1, sida 40

$X = \{X_n; n = 0, 1, \ldots\}$

$E = \{0, 1\}$

$X_n = 0$ innebär att traktorn står stilla dag n

$X_n = 1$ innebär att traktorn fungerar dag n

$$\mathbf{P} = \begin{bmatrix} 0 & 1 \\ 0.1 & 0.9 \end{bmatrix}$$

$$\Pr\{X_{n+1} = 1 | X_n = 0\} = \Pr\{X_1 = 1 | X_0 = 0\}$$

$$= P(0, 1) = 1$$

$$\Pr\{X_{n+1} = 1 | X_n = 1\} = \Pr\{X_1 = 1 | X_0 = 1\}$$

$$= P(1, 1) = 0.9$$

$$\Pr\{X_{n+2} = 1 | X_n = 0\} = \Pr\{X_2 = 1 | X_0 = 0\} = 0.9$$

$$\Pr\{X_{n+2} = 1 | X_n = 1\} = \Pr\{X_2 = 1 | X_0 = 1\} = 0.91$$

$$\mathbf{P}^2 = \begin{bmatrix} 0.1 & 0.9 \\ 0.09 & 0.91 \end{bmatrix}$$

Exempel 2.2, sida 41, 43-44

$X = \{X_n; n = 0, 1, \ldots\}$

$E = \{a, b, c\}$

$X_n = a$ innebär att säljaren är i stad a vecka nr n, etc

$$\mathbf{P} = \begin{bmatrix} 0 & 0.50 & 0.50 \\ 0.75 & 0 & 0.25 \\ 0.75 & 0.25 & 0 \end{bmatrix}$$

$$\mathbf{P}^2 = \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.1875 & 0.4375 & 0.375 \\ 0.1875 & 0.375 & 0.4375 \end{bmatrix}$$

T.ex,

$$\Pr\{X_2 = a | X_0 = b\} = \mathbf{P}^2(b, a) = 0.1875$$

$$\Pr\{X_2 = c | X_0 = b\} = \mathbf{P}^2(b, c) = 0.375$$

$$\Pr\{X_2 = c | X_0 = c\} = \mathbf{P}^2(c, c) = 0.4375$$

$$\mathbf{f} = [\,1000 \quad 1200 \quad 1250\,]^T$$

$$E[f(X_2)|X_0 = a] = \mathbf{P}^2\mathbf{f}(a)$$

$$= [\,0.75 \quad 0.125 \quad 0.125\,]\begin{bmatrix}1000\\1200\\1250\end{bmatrix} \approx 1056.25$$

Antag $\Pr\{X_0 = a\} = 0.5$, $\Pr\{X_0 = b\} = 0.3$

och $\Pr\{X_0 = c\} = 0.2$

Definiera $\mu = [\,0.5 \quad 0.3 \quad 0.2\,]$

$$\Pr_\mu\{X_1 = a\} = 0.375$$

Ekvationen $\pi\mathbf{P} = \pi$ löses av

$$\pi = \tfrac{1}{7}[\,3 \quad 2 \quad 2\,] \approx [\,0.428 \quad 0.286 \quad 0.286\,]$$

$$\lim_n \frac{1}{n}\sum_{i=0}^{n-1} f(X_m) = \frac{7900}{7} \approx 1128.57$$

Exempel 2.5, sida 45-46

NUMBER OF CUSTOMERS	TYPE OF TRADE	
275	Sedan for sedan	$s \to s$
180	Sedan for station wagon	$s \to w$
45	Sedan for convertible	$s \to c$
80	Station wagon for sedan	$w \to s$
120	Station wagon for station wagon	$w \to w$
150	Convertible for sedan	$c \to s$
50	Convertible for convertible	$c \to c$

$E = \{s, w, c\}$

$X_n = s$ innebär att kunden kör en ”sedan” år n, etc

$$\mathbf{P} = \begin{bmatrix} \frac{275}{500} & \frac{180}{500} & \frac{45}{500} \\ \frac{80}{200} & \frac{120}{200} & \frac{0}{200} \\ \frac{150}{200} & \frac{0}{200} & \frac{50}{200} \end{bmatrix} = \begin{bmatrix} 0.55 & 0.36 & 0.09 \\ 0.40 & 0.60 & 0 \\ 0.75 & 0 & 0.25 \end{bmatrix}$$

$$\mathbf{P}^2 = \begin{bmatrix} 0.514 & 0.414 & 0.072 \\ 0.460 & 0.504 & 0.036 \\ 0.600 & 0.270 & 0.130 \end{bmatrix}$$

$$\mathbf{P}^3 = \begin{bmatrix} 0.5023 & 0.4334 & 0.0643 \\ 0.4816 & 0.4680 & 0.0504 \\ 0.5355 & 0.3780 & 0.0865 \end{bmatrix}$$

$$\Pr\{X_{1997} = s, X_{2000} = s | X_{1996} = s\} = P(s,s)P^3(s,s)$$

$$\approx 0.55 \cdot 0.5023 \approx 0.276$$

$$\mathbf{f} = [\,1200 \quad 1500 \quad 2500\,]^T$$

$$E[f(X_{1999})|X_{1996} = s] = E[f(X_3)|X_0 = s] = \mathbf{P}^3\mathbf{f}(s)$$

$$= [\,0.5023 \quad 0.4334 \quad 0.0643\,] \begin{bmatrix} 1200 \\ 1500 \\ 2500 \end{bmatrix} \approx 1413.61$$

Exempel 2.3, sida 41, 48-49, 51-52, 57

$X = \{X_n; n = 0, 1, \ldots\}$

$E = \{1, 2, 3, 4\}$

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.3 & 0.7 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0.2 & 0 & 0.1 & 0.7 \end{bmatrix}$$

$E_1 = \{1\}$, $E_2 = \{2, 3\}$, tillståndet 4 är transient

$\mathbf{P}_1 = [\,1\,]$, $\mathbf{P}_2 = \begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix}$

$\pi \mathbf{P}_2 = \pi \Rightarrow \pi = \frac{1}{12}[\,5 \quad 7\,] \approx [\,0.417 \quad 0.583\,]$

$$\mathbf{R} = \begin{bmatrix} \infty & 0 & 0 & 0 \\ 0 & \infty & \infty & 0 \\ 0 & \infty & \infty & 0 \\ \infty & \infty & \infty & \frac{10}{3} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{7}{10} \end{bmatrix}$$

Exempel 2.8, sida 51

$E = \{1, 2, s, g\}$

$$\mathbf{P} = \begin{bmatrix} 0.2 & 0.7 & 0.1 & 0.0 \\ 0.05 & 0.1 & 0.05 & 0.8 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Tillstånden $1, 2$ är transienta

Tillstånden s, g är absorberande

Irreducibla klasser är följaktligen

$E_s = \{s\}$, $E_g = \{g\}$

Restriktionen av $\mathbf{P}$ till de transienta tillstånden $1, 2$ är

$$\mathbf{Q} = \begin{bmatrix} 0.2 & 0.7 \\ 0.05 & 0.1 \end{bmatrix}$$

$$\mathbf{b}_s = \begin{bmatrix} 0.10 \\ 0.05 \end{bmatrix}$$

$$\mathbf{R} = (\mathbf{I} - \mathbf{Q})^{-1} \approx \begin{bmatrix} 1.3139 & 1.0219 \\ 0.0730 & 1.1679 \end{bmatrix}$$

$$(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{b}_s \approx \begin{bmatrix} 0.1825 \\ 0.0657 \end{bmatrix}$$

Så $F(1, s) \approx 0.1825$

$$\mathbf{b}_g = \begin{bmatrix} 0.0 \\ 0.8 \end{bmatrix}$$

$$(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{b}_g \approx \begin{bmatrix} 0.8175 \\ 0.9343 \end{bmatrix}$$

Så $F(1, g) \approx 0.8175$

Exemple 2.13, sida 62-63

(forts av exempel 2.8)

Cost of raw material: $c_r = 150$ per part

Processing cost: $f(1) = 200$, $f(2) = 300$ each time a part is processed in Step 1 and 2, resp

Disposal cost: $c_s = 50$ per part sent to scrap

$$C = c_r + f(1)N_1 + f(2)N_2 + c_s I_{\{T^s < \infty\}}$$

$$C = 150 + 200N_1 + 300N_2 + 50 I_{\{T^s < \infty\}}$$

$$E[C|X_0 = 1] = 150 + 200R(1,1) + 300R(1,2) + 50F(1,s)$$

$\mathbf{R} = (\mathbf{I} - \mathbf{Q})^{-1}$ ger oss

$R(1,1) \approx 1.3139$, $R(1,2) \approx 1.0219$

Vi vet redan att $F(1,s) \approx 0.1825$

Så $E[C|X_0 = 1] \approx 728.47$