Axioms of probability

1. Let S denote the sample space for an experiment. Then
$$P[S] = 1$$
2. Let A be an event. Then
$$P[A] \geq 0$$
3. Let $A_1, A_2, A_3, \ldots$ be a finite or an infinite collection of mutually exclusive events. Then
$$P[A_1 \cup A_2 \cup \cdots] = P[A_1] + P[A_2] + \cdots$$

Notera skrivsätten
$$A_1 \cup A_2 \cup \cdots = \bigcup_i A_i$$
och
$$A_1 \cap A_2 \cap \cdots = \bigcap_i A_i$$
Jämför med
$$P[A_1] + P[A_2] + \cdots = \sum_i P[A_i]$$

Theorem 2.1.1:
$$P[\emptyset] = 0$$

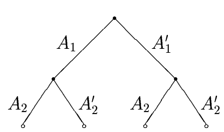
Theorem 2.1.2:
$$P[A'] = 1 - P[A]$$

General addition rule:
$$P[A_1 \cup A_2] = P[A_1] + P[A_2] - P[A_1 \cap A_2]$$

Definition 2.2.1: Conditional probability

Let A_1 and A_2 be events such that $P[A_1] > 0$. The conditional probability of A_2 given A_1, denoted by $P[A_2|A_1]$, is defined by
$$P[A_2|A_1] = \frac{P[A_1 \cap A_2]}{P[A_1]}$$

Notera: $P[A_1 \cap A_2] = P[A_1]P[A_2|A_1]$



Theorem 2.4.1: Bayes' theorem

Let $A_1, A_2, \ldots, A_n$ be a collection of mutually exclusive events whose union is S. Let B be an event such that $P[B] > 0$. Then for any of the events A_j, $j = 1, 2, \ldots, n$,
$$P[A_j|B] = \frac{P[A_j]P[B|A_j]}{\sum_{i=1}^{n} P[A_i]P[B|A_i]}$$

Notera att
$$P[B] = \sum_{i=1}^{n} P[A_i]P[B|A_i]$$

Definition 2.3.1: Independent events

Events A_1 and A_2 are independent if and only if

$$P[A_1 \cap A_2] = P[A_1]P[A_2]$$

Notera: A_1, A_2 är oberoende $\Leftrightarrow P[A_2|A_1] = P[A_2]$ (sats 2.3.1)

Definition 2.3.2:

A finite collection $A_1, A_2, \ldots, A_n$ of events are independent if and only if, given any subcollection $A_{(1)}, A_{(2)}, \ldots, A_{(m)}$,

$$P[A_{(1)} \cap A_{(2)} \cap \cdots \cap A_{(m)}] = P[A_{(1)}]P[A_{(2)}] \cdots P[A_{(m)}]$$

Notera skrivsättet

$$x_1 \cdot x_2 \cdots x_n = \prod_{i=1}^{n} x_i$$

Definition: Oberoende försök

Två försök är oberoende om det för varje händelse A_1 i det ena försöket och varje händelse A_2 i det andra, gäller att A_1 och A_2 är oberoende. Att tre eller fler försök är oberoende definieras analogt.