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1. Probability rules

Random experiments and random events
- sample space, complementary event
- intersection, union, Venn diagrams

Probability = population proportion
- rule 1: division rule
- rule 2: addition rule

Conditional probability and independent events
- rule 3: multiplication rule

Partition of the sample space
- rule 4: LTP
- rule 5: Bayes' rule

2. Random variables

Probability distribution functions:
- pmf, pdf, cdf

Distribution center and spread:
- mean, median, mode; variance, standard deviation

Discrete distributions:
- dU(N), Bin(n, p), Hg(N, n, p), Geom(p), Pois(λ)

Continuous distributions:
- U(a, b), Exp(λ), N(μ, σ^2)

Central Limit Theorem: $\bar{X} \in \mathrm{N}(\mu, \frac{\sigma^2}{n})$
- normal approximations for Bin, Pois, Hg

3. Joint distributions

Joint pmf and pdf
  marginal and conditional distributions
  independent random variables
Conditional expectation: LTE and LTV
  optimal predictor and optimal linear predictor
Association between two random variables
  covariance and correlation coefficient
  uncorrelated random variables
  addition rule of variance
Multinomial and Bivariate normal distributions

4. Parameter estimation

Types of data:
  continuous, discrete, categorical, dichotomous
Population parameters and sample parameters
  sample mean and sample variance
  point and interval estimates
Sampling distribution and sampling error:
  systematic and unsystematic errors
  unbiased and consistent estimates
  estimated standard error $s_{\bar{X}}=\frac{s}{\sqrt{n}}$, $s_{\hat{p}}=\sqrt{\frac{\hat{p}\hat{q}}{n-1}}$
Sampling with and without replacement
  finite population correction
Parametric statistical models, MME and MLE

Confident intervals (CI) exact and approximate
  prediction interval (PI) for a new observation
  t-distribution, degrees of freedom

5. Hypotheses testing

Statistical hypotheses
  simple and composite
  one-sided and two-sided
Test statistic, rejection region
  exact and approximate null distributions
Two types of error
  conflict between two error sizes α and β
  significance level and P-value (one- and two-sided)
Small sample tests
  one-sample t-test, normal population distribution
  the small-sample test for the proportion
Large-sample tests for proportion and for mean
  duality of CI and hypotheses testing
The power of the test, planning of sample size

6. Simple linear regression

Association between two continuous variables
  sample correlation coefficient
Simple linear regression model
  least square estimates

Regression line
  decomposition of the total sum of squares
  coefficient of determination
CI and hypothesis testing concerning the slope β_1
  model utility test of H_0: $\beta_1 = 0$
Prediction of a future value Y_{n+1} and its mean
  given the new independent variable value x_{n+1}

7. Chi-square tests

Fitting a parametric model to the data
  Pearson's chi-square test
  chi-square distirbution
Association between two discrete or categorical variables
  χ^2-test of independence (one cross-classified sample)
  χ^2-test of homogeneity (several independent samples)
  grouping together of smaller cells

8. Decision theory and Bayesian inference

Risk function for a decision rule based on a loss function
  minimax decision rules
Bayesian approach: prior and posterior distributions
  posterior risk and Bayes action
Conjugate priors
  Beta and Dirichlet distributions, pseudocounts
  Gamma distribution

Bayesian estimation
  MAP and 0-1 loss function
  PME and squared error loss function
  credibility interval
Bayesian hypotheses testing

List of relevant problems in the book

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Chapter 8: 7, 26, 27, 46, 47
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  2, 5, 14, 15, 18b-f, 19, 20
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