## HOMEWORK 1

Let  $X_i, i = 1, ..., n$  be an iid sample from a  $U[a\theta, b\theta]$ , where  $\theta > 0$  and 0 < a < b

- (a) Find a sufficient statistic for  $\theta$ .
- (b) Find the minimum sufficient statistic.
- (c) Find the MLE for  $\theta$ . Explain how you decide it is the MLE. (If you have a difficult time with this,
- try plotting the likelihood for simulated data for chosen a and b.)

(d) Come up with a Method of Moments estimator for  $\theta$ .

(e) Is the MLE unbiased? (Hint: derive the distribution for maximums.)

(f) Construct a simulation study that compares the MoM and the MLE for different sample sizes and true values of  $\theta$ . Do the absolute values of a and b matter? or their relative value (the range)?

(g) In the special case, a = 0, b = 1, derive the bias and estimation variance for the MLE and MoM?