MSG800/MVE170 Basic Stochastic Processes Fall 2013

Exercise Session 2

Equations (equation numbers) and problems (problem numbers) in Chapter 5 of the Second Edition of Hsu's book do not agree completely with those in the First Edition: The difference is that the Second Edition contains a new Section 5.8 about martingales as compared with the First Edition that has been inserted before the problems (exercises). As a result of that additional corresponding problems also have been added to the Second Edition.

In numbers: Equations 5.1-5.65 and Problems 5.1-5.62 agree in both editions of the book (except that equation numbers in the problems in the Second Edition are 23 units higher that in the First Edition due to the new Section 5.8). Equations 5.66-5.88 in Section 5.8 and Equations 5.208-5.226 in the corresponding set of problems of the Second Edition are not present in the First Edition, while Equations 5.89-5.207 in the Second Edition correspond to Equations 5.66-5.184 in the First Edition. Problems 5.63-5.82 and Supplementary Problems 5.103-5.106 in the Second Edition are not present in the First Edition, while Supplementary Problems 5.83-5.102 in the Second Edition correspond to Supplementary Problems 5.63-5.82 in the First Edition.

Sections 5.6-5.8 in Hsu's book

Solved problems. Problems 5.49, 5.55, 5.60, 5.72, 5.76, 5.77, 5.78 and 5.81 in the Second Edition of Hsu's book (as mentioned above Problems 5.72, 5.76, 5.77, 5.78 and 5.81 are not available in the First Edition).

Problems for own work. Problems 5.98, 5.100, 5.101, 5.104 and 5.105 in the Second Edition of Hsu's book (as mentioned the first three of these correspond to Problems 5.78, 5.80 and 5.81 in the First Edition, while the last two are not available in the First Edition).

Computer problem. A Nevada own-home-owner bicycles to Las Vegas in order to bet all his money \$100 on a \$1 slot machine hoping to end up doubling his capital to make it possible for him to pay a pending \$200 mortage on his own-home. The slot machine game process is a time discrete stochastic process $M_n = \sum_{i=1}^{n} X_i$ for $n \in \mathbb{N}$, where $M_0 = 100$

is the initial capital and $\{X_i\}_{i=1}^{\infty}$ is a sequence of independent identically distributed random variables with $\mathbf{P}\{X_i = 4\} = 1/5$ and $\mathbf{P}\{X_i = -1\} = 4/5$ describing the gain or loss at the consecutive pulls of the slot machine. The own-home-owner carries on the slot machine game until the first time $T = \min\{n \geq 1 : M_n \geq 200 \text{ or } M_n = 0\}$ at which either the capital is doubled (in which case he bicycles home and pays his mortage) or the capital is depleted (in which case he bicycles home to sell his own-home).

The process $\{M_n\}_{n=0}^{\infty}$ is a martingale and the random time T is a stopping time that satisfy the conditions of the optimal stopping theorem. (It is a very useful exercise for the extra ambitious student to prove that this is really so.) Writing p for the probability that the own-home-owner can pay his mortage it follows from the optional stopping theorem that

$$100 = \mathbf{E}\{M_0\} = \mathbf{E}\{M_T\} \begin{cases} \leq p \cdot 203 \\ \geq p \cdot 200 \end{cases},$$

so that $p \in [\frac{100}{203}, 1/2]$. Find by means of computer simulations a better estimate of the value of the probability p than that obtained in this manner from the optional stopping theorem.