Partial differential equation F - (alt.) project 1

Goal: To seek a qualitative understanding of Poisson type equations, with terms modelling diffusion, convection and production/reaction/absorbtion type processes, and to study the interaction of these terms/processes. Further, to realize that derivatives is nothing but continuous models of local couplings on a discrete level, e.g. of nodal values.

Computer lab work

1. Run the demo solver `MySvenssonSolver.m` (see home page), for Laplace equation on a given rectangular domain, with given boundary values $g(x)$. Study the code carefully to fully understand what it is doing. Change the given boundary values to check that the solver works properly.

2. Modify the code so that it can solve also Poissons equation, with given right hand side load or production term f.

3. Modify further so that the equation may include a) a "zero order" term of the form $a\,u$, with $a(x)$ a given function, and b) a variable diffusivity/conductivity d, replacing the Laplace operator by $\operatorname{div}(d\,\operatorname{grad} u)$. In particular, study the effect of letting the diffusivity tend to zero (with $a = 1$, $f = x$ and $g = 0$, say).

4. Solve some nonlinear problem with $f = f(u)$.

5. Add a convection term of the form $c \cdot \operatorname{grad} u$. In particular, study the effect of this term as d decreases (with $c = (-1, 0)$, $a = 0$, and $f = x$, say).

6. If possible, seek ways to implement boundary conditions of the form $n \cdot d\,\operatorname{grad} u = k\,(g - u)$, with $k = 0$ to begin with. Hint: see `MySvenssonSolver3.m`

7. If possible, seek to generalise the method to non-rectangular domains. Hint: see `MySvenssonSolver2.m`. This code may need to be modified to work for non-homogeneous boundary data.

8. As an alternative to solving the resulting set of coupled equations through an iterative procedure, set up the system as a matrix equation and solve by Gaussian elimination (using matlabs \\).

9. Generalize to some problem in 3D. Note that matlab can handle also "3D-matrices".

10. Interprete the iterative solution process as the solution of a time dependent problem!