Lösningar Tillämpad Matematik TMA990 Kb2 010822

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1. b) Fourierkoefficienterna ges av

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{2\pi^2}{3};$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx dx = \frac{4(-1)^n}{n^2}, \quad n > 0;$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \sin nx dx = 0, \quad n > 0;$$

Vi får

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx.$$

c) $x = \pi$ ger

$$VL = \pi^2 \text{ och } HL = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

vilket ger att

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

2. (a) Se tex Petersson: Fourieranalys.

(b) $(f * f')(t)$ har Fouriertransform $\hat{f}(\xi) i\xi \hat{f}(\xi)$. Plancherels formel ger

$$\int_{-\infty}^{\infty} |(f * f')(t)|^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\hat{f}(\xi) i\xi \hat{f}(\xi)|^2 \, d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} \xi^2 |\hat{f}(\xi)|^4 \, d\xi.$$

Vi får

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \xi^2 |\hat{f}(\xi)|^4 \, d\xi = \frac{1}{\pi}\int_{0}^{\infty} \xi^2 (\xi^3 + 1)^{-4} \, d\xi = \frac{1}{9\pi}.$$

3. Inhomogen ekvation så vi sätter

$$u(x,y) = v(x,y) + S(x)$$

Vi löser först

$$S''(x) = x, \quad S'(0) = 0, \quad S(1) = 0$$

och får

$$S(x) = \frac{x^3 - 1}{6}$$

Kvar att lösa har vi ekvationen

$$v_{xx} + v_{yy} = 0, \quad v_x(0, y) = 0, \quad v(1, y) = ye^{-|y|}.$$

Fouriertransformering i y-led ger

$$\hat{v}_{xx}(x, \omega) - \omega^2 \hat{v}(x, \omega) = 0, \quad \hat{v}_x(0, \omega) = 0, \quad \hat{v}(1, \omega) = -\frac{4i\omega}{(1+\omega^2)^2}.$$

Alllmän lösning

$$\hat{v}(x, \omega) = A(\omega) \cosh(x\omega) + B(\omega) \sinh(x\omega).$$

Data ger

$$\hat{v}_x(0, \omega) = 0 \Longrightarrow B(\omega) = 0.$$

$$\hat{v}(1, \omega) = -\frac{4i\omega}{(1+\omega^2)^2} = A(\omega) \cosh(\omega).$$

Fouriers inversionsformel ger nu

$$v(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -\frac{4i\omega \cosh(x\omega)}{(1+\omega^2)^2 \cosh(\omega)} e^{iy\omega} d\omega =$$

$$\frac{4}{\pi} \int_0^{\infty} -\frac{\omega \cosh(x\omega) \cosh(y\omega)}{(1+\omega^2)^2 \cosh(\omega)} d\omega.$$

Summering av S och v ger nu u.