PDE Project Course 05/06

Introduction

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Computational Technology

Welcome

Teachers:

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Course page:
Follow the links education → courses, starting at

- `http://www.math.chalmers.se/cm/`

Partial differential equations (PDE)

Solve

$$A(u) = f$$

where A is a differential operator, f is a given force term and u is the solution.
Important questions:

- Existence/uniqueness of solutions
- Computation of solutions

ACMM

Overall goal of Automation of Computational Mathematical Modeling (ACMM): build a computational machine which takes any PDE and a tolerance for the error, automatically computes a solution.
Can be broken down into several sub-tasks:

1. Error estimation: Automatically generate an error estimate for a given model.
2. Adaptivity: Automatically choose a discrete model that satisfies the error estimate.
3. Assembly: Automatically generate a discrete system (equation system for degrees of freedom) given a discrete model.

Contents of the course

- Computation of solutions to PDEs
- Finite Element Method
- Mostly implementation, not so much theory
- Programming (Python/C++)
- Independent work
- Presentations of results and report writing

Prerequisites

- PDE/FEM course
- Some knowledge of programming

Examples of PDE

- $A(u) = -\Delta u = f$ Poisson's equation
- $A(u) = \dot{u} - \Delta u = f$ The heat equation
- $A(u) = \ddot{u} - \Delta u = f$ The wave equation

These are the main examples of linear *elliptic*, *parabolic*, and *hyperbolic* PDE.

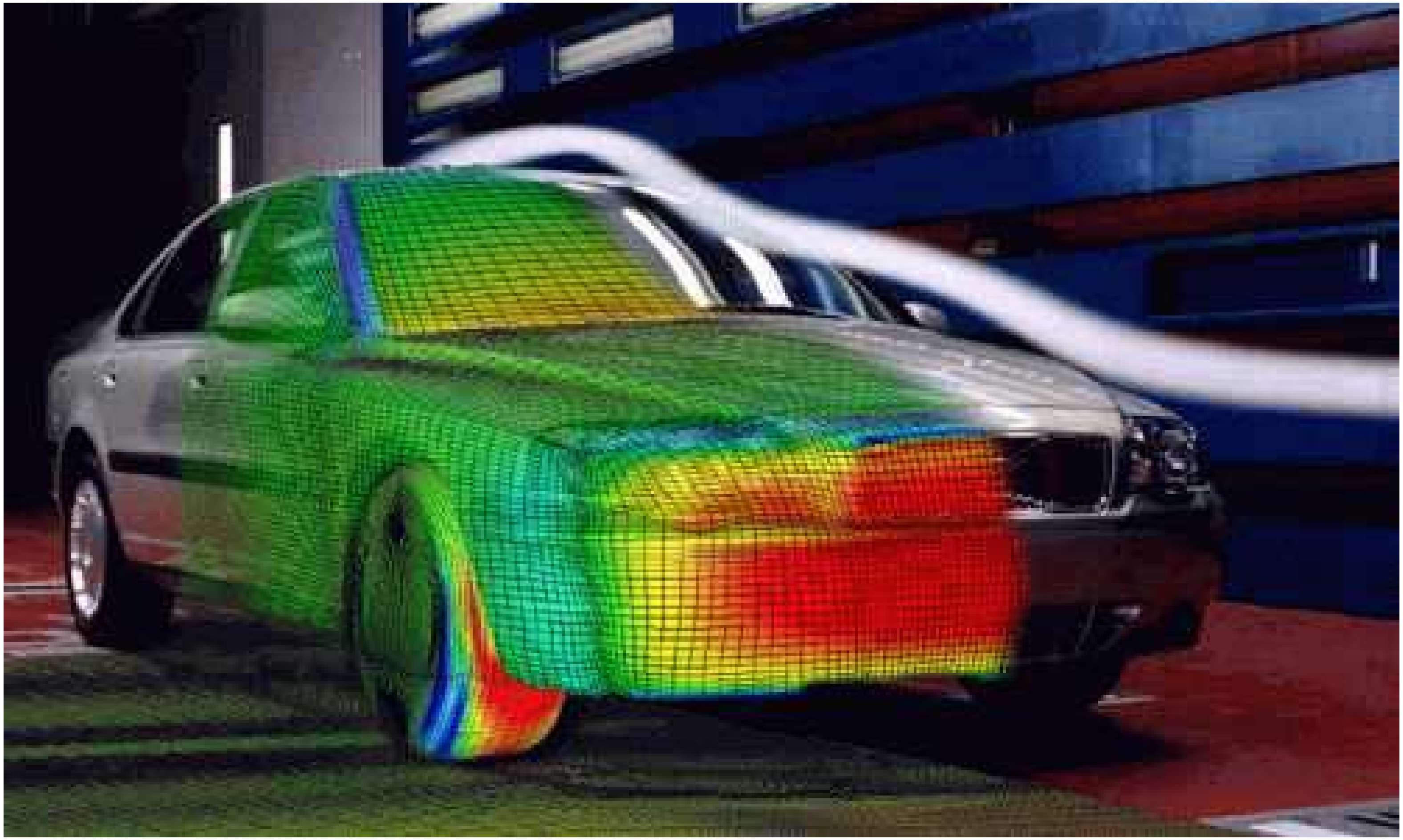
Examples of PDE

The Navier–Stokes equations:

$$A(u) = \begin{pmatrix} \dot{v} + v \cdot \nabla v - \nu \Delta v + \nabla p \\ \nabla \cdot v \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix}$$

where the solution $u = (v, p)$ consists of the the fluid velocity v and the pressure p.

Examples of PDE

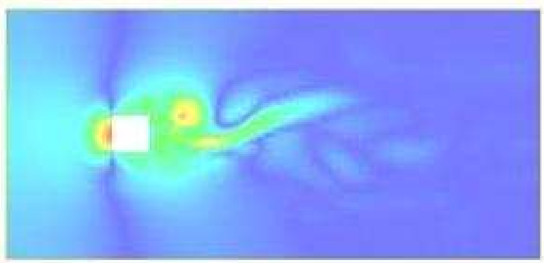


Numerical solution of the Navier-Stokes equations.

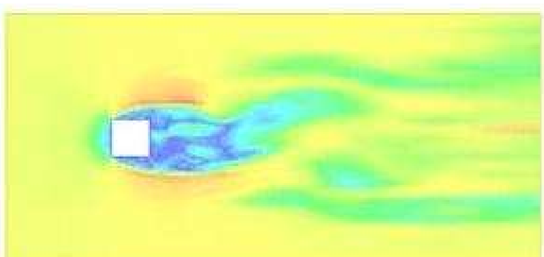
Examples of PDE

Turbulent flow around a surface mounted cube:

Pressure:



Norm of velocity:



Examples of PDE

The equations of linear elasticity:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - v &= 0 \quad \text{in } \Omega^0, \\
\frac{\partial v}{\partial t} - \nabla \cdot \sigma &= f \quad \text{in } \Omega^0, \\
\sigma &= E\epsilon(u) = E(\nabla u^\top + \nabla u) \\
v(0,\cdot) &= v^0, \quad u(0,\cdot) = u^0 \quad \text{in } \Omega^0.
\end{aligned}
$$

Only valid for small displacements.

Examples of PDE

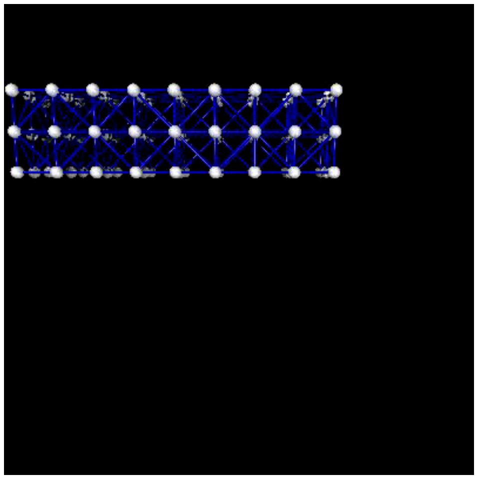
Updated Lagrangian formulation:

$$
\begin{aligned}
\frac{\partial u}{\partial t} - v &= 0 \quad \text{in } \Omega(t), \\
\frac{\partial v}{\partial t} - \nabla \cdot \sigma &= f \quad \text{in } \Omega(t), \\
\frac{\partial \sigma}{\partial t} &= E\epsilon(v) = E(\nabla v^\top + \nabla v) \\
v(0,\cdot) &= v^0, \quad u(0,\cdot) = u^0 \quad \text{in } \Omega^0.
\end{aligned}
$$

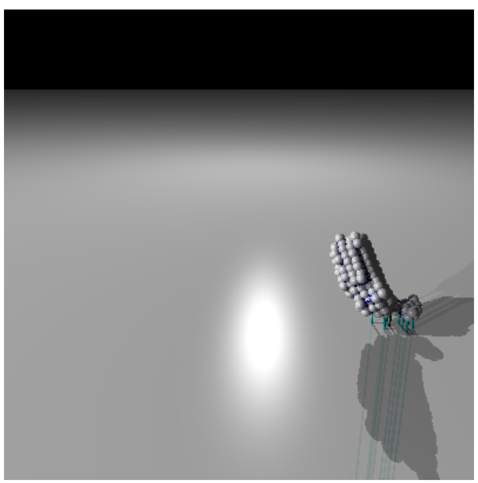
Also valid for large displacements.

Examples of PDE

Elastic beam (rubber) under gravity:



Elastic cow with contact:



Literature

PDE/FEM:

- *Applied Mathematics: Body and Soul*, by Eriksson, Estep, and Johnson, Springer Verlag 2003
- *Computational Differential Equations*, by Eriksson, Estep, Hansbo, and Johnson. Studentlitteratur 1996

Books and material

Programming:

- *C++ Primer*, by Lippman. Addison-Wesley 1995 (Old but quite good)
- *The C++ Programming Language*, by Stroustrup. Addison-Wesley 1997 (Not for beginners)
- *Python Reference Manual*, by Guido van Rossum. `www.python.org`

Resources on the web

- *Body and Soul computer sessions*:
  www.phi.chalmers.se/bodysoul/sessions/

Software Tools

- **FEniCS**
- DOLFIN/PyDOLFIN
- **FEniCS** form compiler (FFC)
- FemLab (mesh generation/visualisation)
- ParaView/Mayavi (visualisation in 3D)
- Triangle (mesh generation in 2D)
- TetGen (mesh generation in 3D)

Lecture plan

1. *Implementation of finite element methods*
2. ***FEniCS**: FFC and DOLFIN*
3. *Python tutorial*

Schedule

For a detailed schedule look at the homepage.
Lectures:

- Mondays 13-15 in MV-F24
- Fridays 10-12 in MV-F24

Consultation:

- e-mail preferred
- Mondays 13-15
- Fridays 10-12

Deadlines

Project plan:

- Friday January 27 at 17.00

Progress reports:

- course week 4 and 6

Presentations:

- course week 9

Project report:

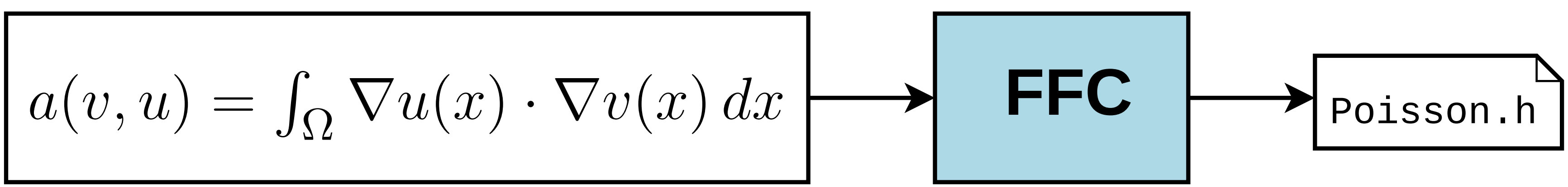
- Friday March 17 at 17.00

DOLFIN

- Numerical solution of PDE using FEM
- 2D / 3D
- Object-oriented (C++)
- Python interface (PyDOLFIN)
- GPL licence

FFC

- Form compiler - takes form and element as input, generates assembly code
- 2D / 3D
- Object-oriented (Python)
- Generates code in several formats, primarily DOLFIN format is used
- GPL licence



Puffin

- Numerical solution of PDE using FEM
- 2D
- Written for Matlab (Octave)
- GPL licence

Project / Examination

- Groups of max 2 students
- Submit a project plan
- Two mandatory oral progress reports
- Presentation
- Written report

Project plan

- Easy PDE - write solver from scratch
- Advanced PDE - use existing tools (if you want)
- State what grade the group is aiming for

Project ideas

- ...