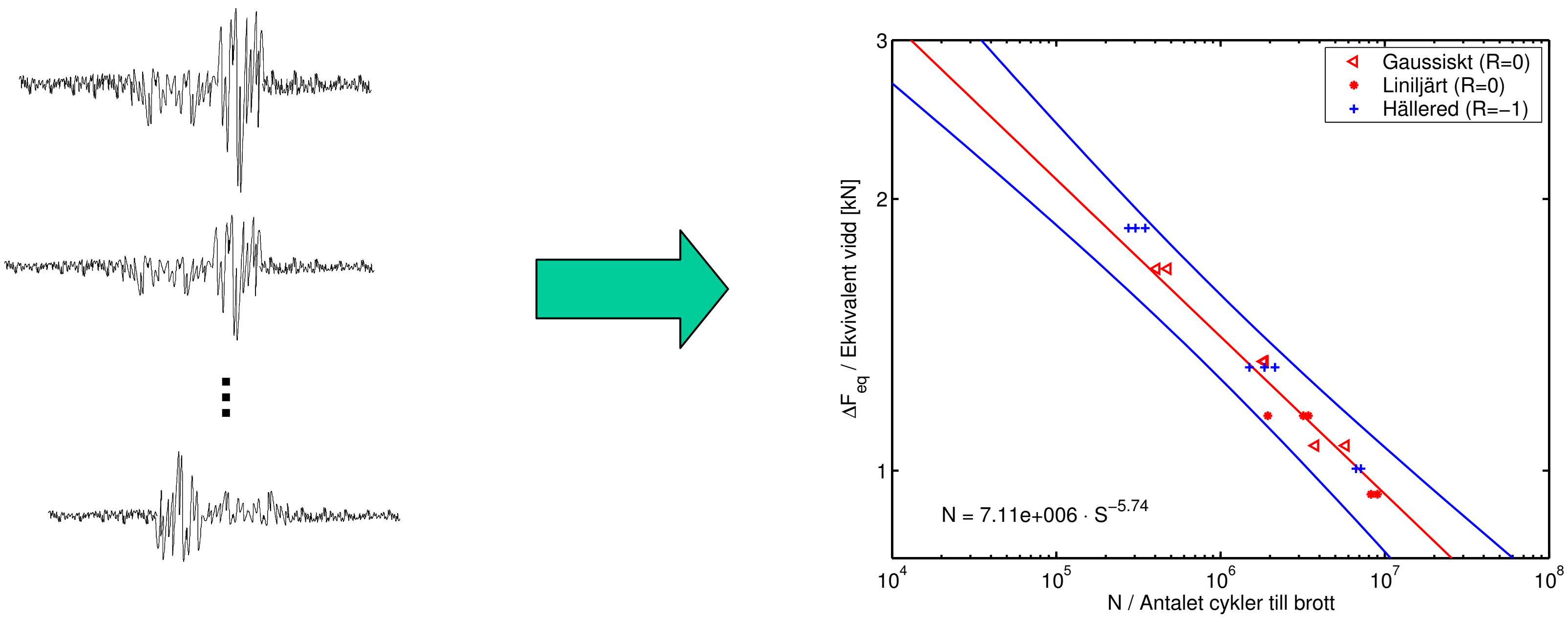
Fatigue Life Prediction Based on Variable Amplitude Tests



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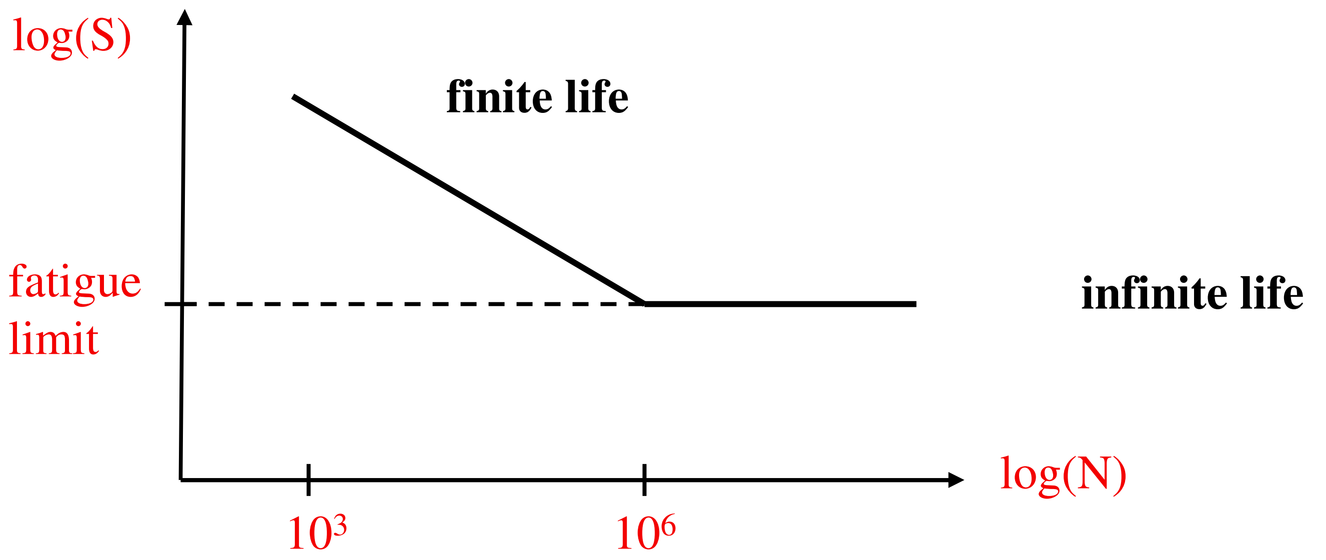
Outline

- What is metal fatigue?
- Life Prediktion & Testing – Traditional Method
- Life Prediktion & Testing – Proposed Method
- Examples
- Extensions of the model
- Conclusions

What is Fatigue?

- **Fatigue is the phenomenon that a material gradually deteriorates when it is subjected to repeated loadings.**

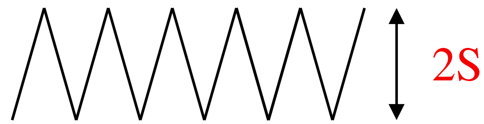
- **Wöhler (1858), Railway engineer**
- **Model for fatigue life: Wöhler-curve**
  number of cycles N to failure as function of cycle amplitude S.



Fatigue Life, Load Analysis, and Damage

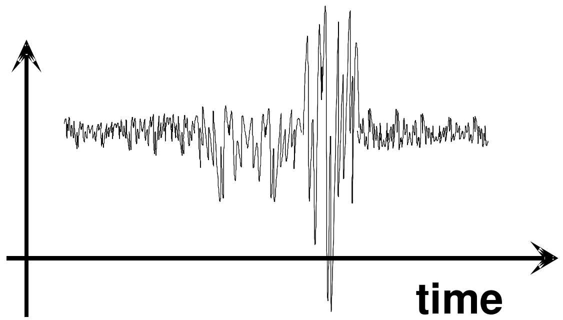
- **SN-curve:**

$$N = \alpha S^{-\beta}$$



  - *α, β material parameters.*

- **Cycle counting**
  - *Convert a complicated load function to equivalent load cycles.*
  - *Load $X(t)$ gives amplitudes S_1, S_2, S_3, …*



- **Palmlgren-Miner damage accumulation rule**
  - *Each cycle of amplitude S_i uses a fraction $1/N_i$ of the total life.*
  - *Damage i time [0,T]:*

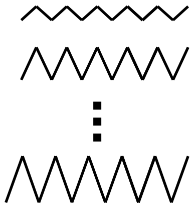
$$D_T = \sum_i \frac{1}{N_i} = \alpha^{-1} \sum_i S_i^{\beta}$$

  - *Failure occurs when all life is used, i.e when D>1.*

Life Prediktion & Testing – Traditional Method

Fatigue testing

CA
Constant
Amplitude

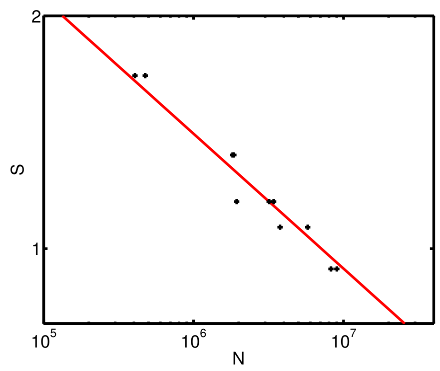


Analysis

- **Basquin**

$$N = \alpha S^{-\beta}$$

Wöhler-curve

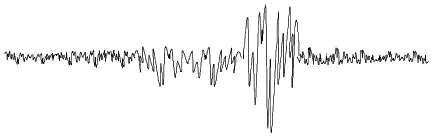


Prediktion

- **Palmgren-Miner**
- **Rainflow count**
- **Basquin**

Predicted life

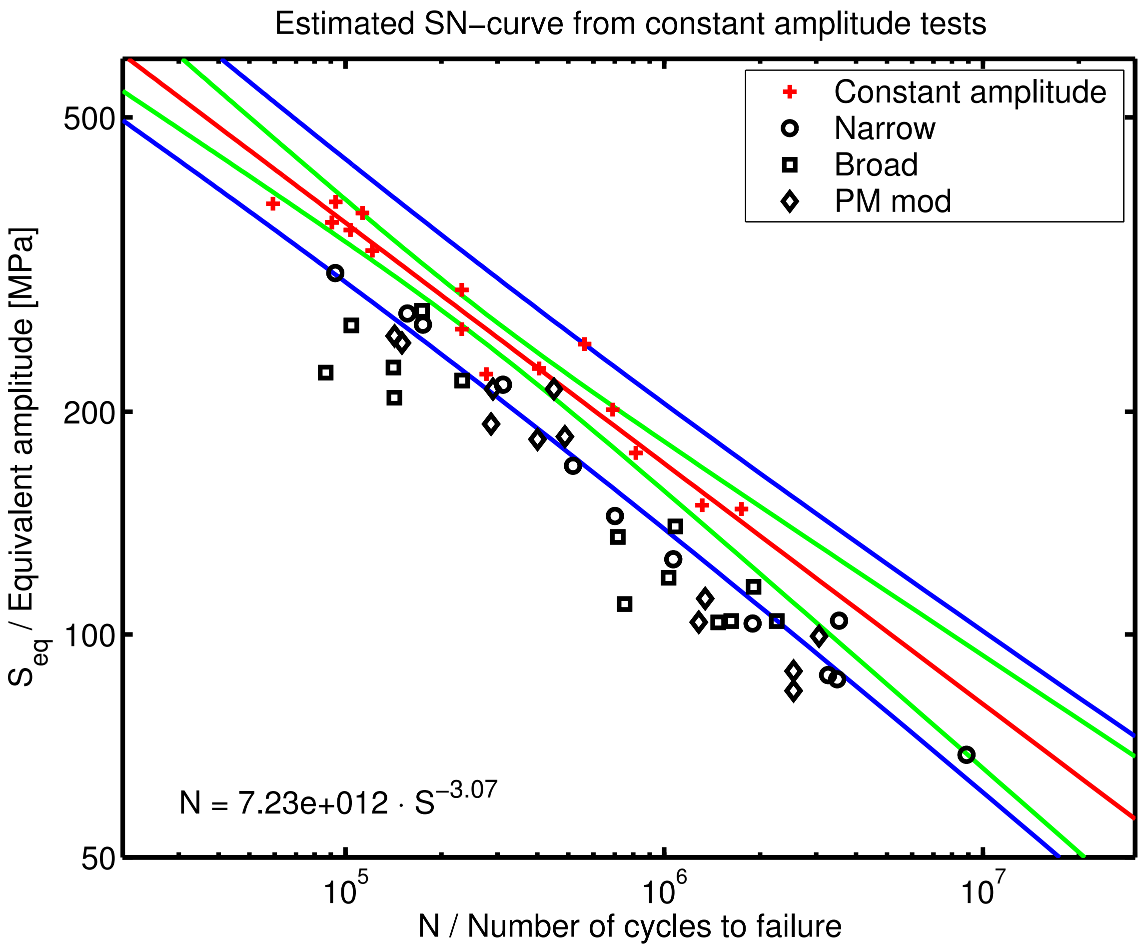
VA
Variable Amplitude



Disadvantages: **Often systematic prediction errors. Model errors!**
Could depend on sequence effects, residual stresses, and threshold effects.

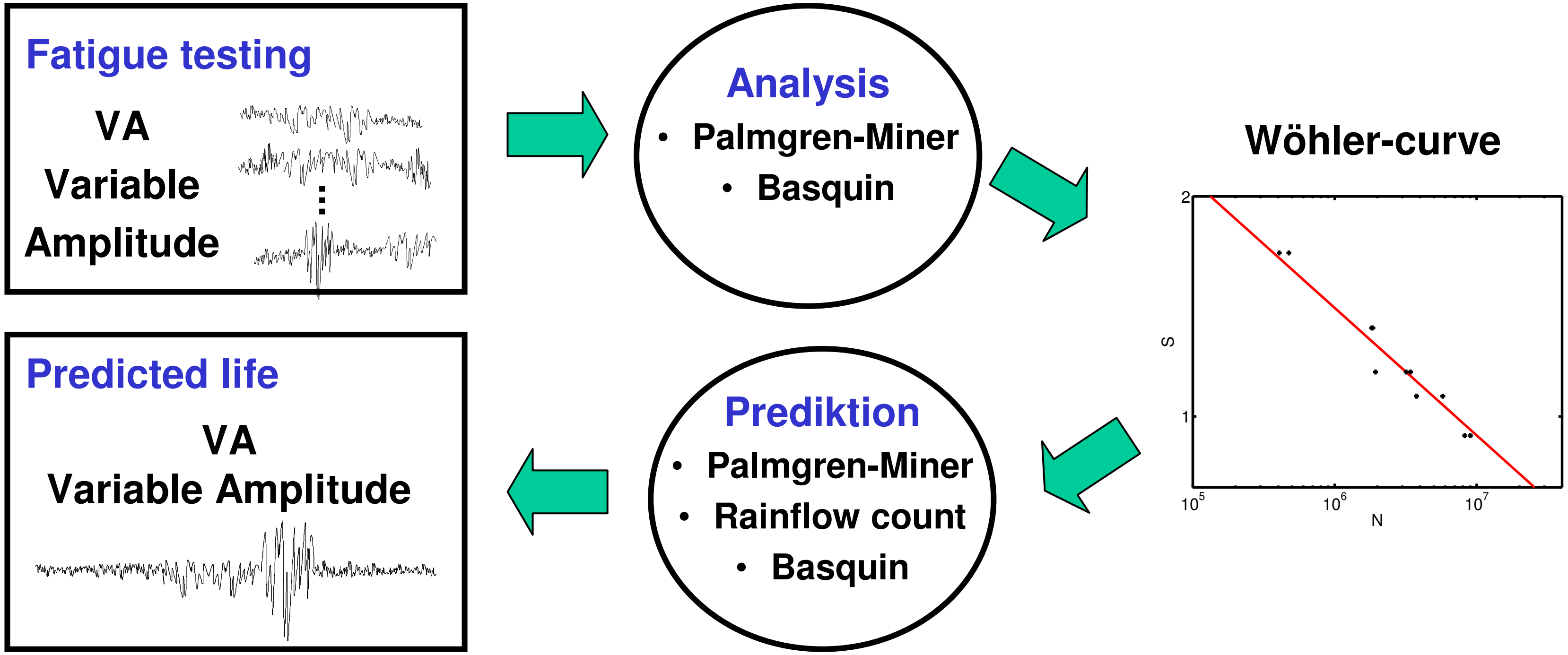
Empirical correction: **Change damage criterion, e.g.** ~~D = 1~~ **D = 0.3**

Example: Agerskov Data Set



- **Welded steel, non load carrying weld.**
- **Estimation from CA: Linear regression.**
- **Here it gives non-conservative predictions.**
- **Solution suggested by Schütz (1972): Relative Miner-rule. Calculate a correction factor based on VA tests, i.e. change α but keep β fixes.**

Life Prediktion & Testing – Proposed Methodology



Advantage: Same load type at both testing and prediction.
Should reduce possible model errors, and give better predictions.

Inspiration: - Gassner-line (1950) - Relativ Miner (1972) - Omerspahic (1999)

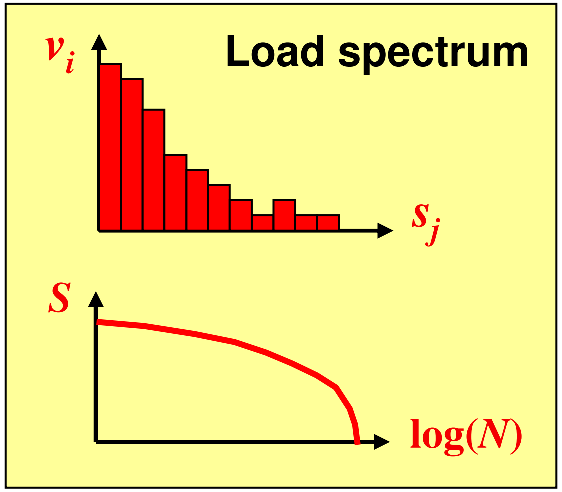
Problem Description

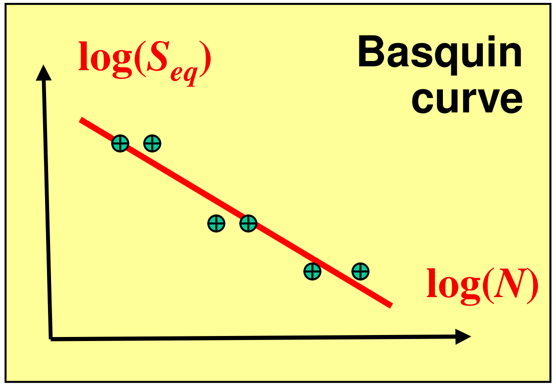
Model

- *SN-curve, Basquin:* $N_i = \alpha S_i^{-\beta} \cdot e_i$
- *A VA load is specified through a load spectrum, i.e. the frequencies v_j of the load amplitudes s_j, $L=(s_j,v_j)$*
- *An equivalent load amplitude is defined as*

Equivalent Amplitude: $S_{eq} = \sqrt[\beta]{\sum v_j s_j^{\beta}}$

- *Damage equivalent to load spectrum. (Palmgren-Miner damage accumulation)*





Estimation

- *Maximum Likelihood ↔ non-linear regression.*
- *Uncertainty in estimates.*
- *Uncertainty in prediction.*

Estimation of Wöhler curve for variable amplitude loads

Condense the VA load to a spectrum ψ ***of counted load cycles and define its equivalent load amplitude*** S_{eq}

$$\psi = \{ \nu_k , s_k ; i = 1,2,\ldots m \} \qquad S_{eq} = \left(\sum_{i=1}^{m} \nu_i S_a^{\beta} \right)^{1/\beta}$$

Formulate the logarithm of the Basquin equation $N_i = \alpha S_i^{-\beta} \cdot e_i$ ***...***

$$\ln N_i = \ln \alpha - \beta \ln S_{eq,i} + \varepsilon_i = f(\psi_i ; \alpha, \beta) + \varepsilon_i \qquad \varepsilon_i = \ln e_i \in N(0, \sigma^2)$$

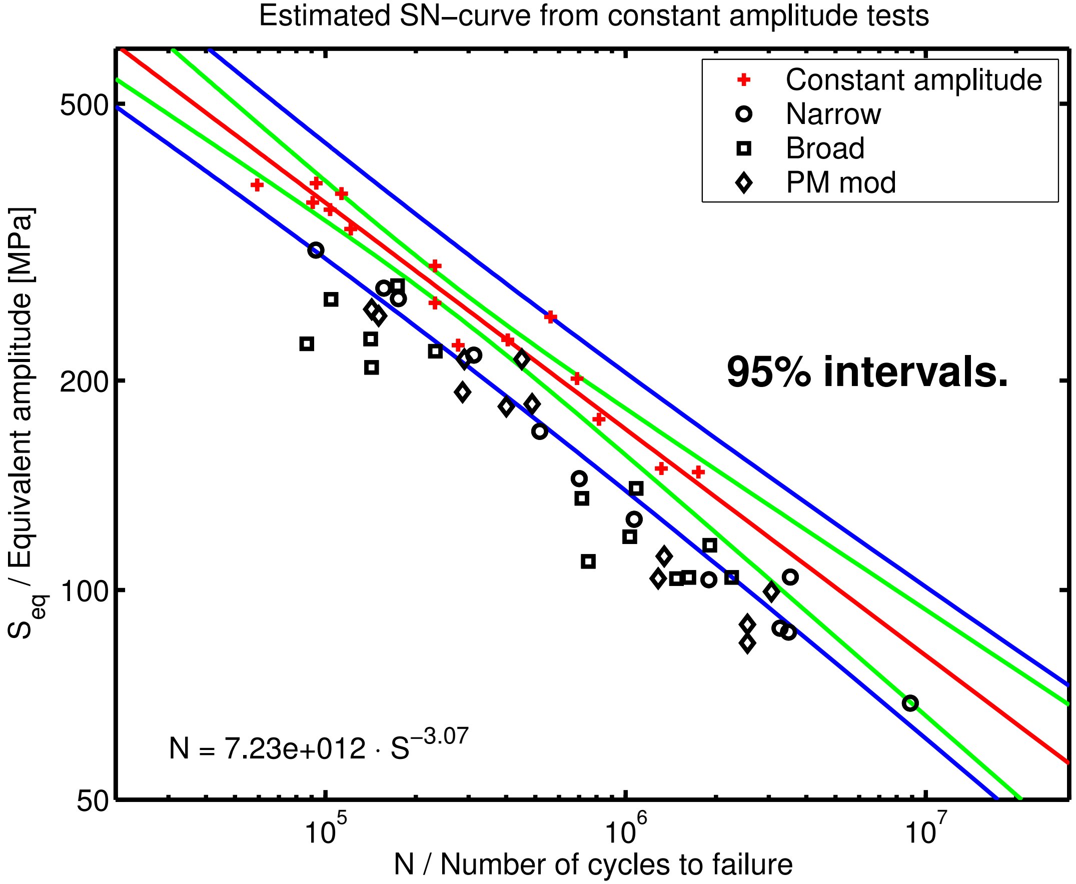
… and ML estimate of the parameters from n reference spectrum tests

$$(\hat{\alpha}, \hat{\beta}) = \underset{(\alpha,\beta)}{\arg\min} \sum_{i=1}^{n} \left[\ln N_i - f(\psi_i ; \alpha, \beta) \right]^2$$

$$\hat{\sigma}^2 = s^2 = \frac{1}{n-2} \sum_{i=1}^{n} \left[\ln N_i - f(\psi_i ; \hat{\alpha}, \hat{\beta}) \right]^2$$

Example: Agerskov

Estimated SN-curve from CA-tests, predictions for VA.

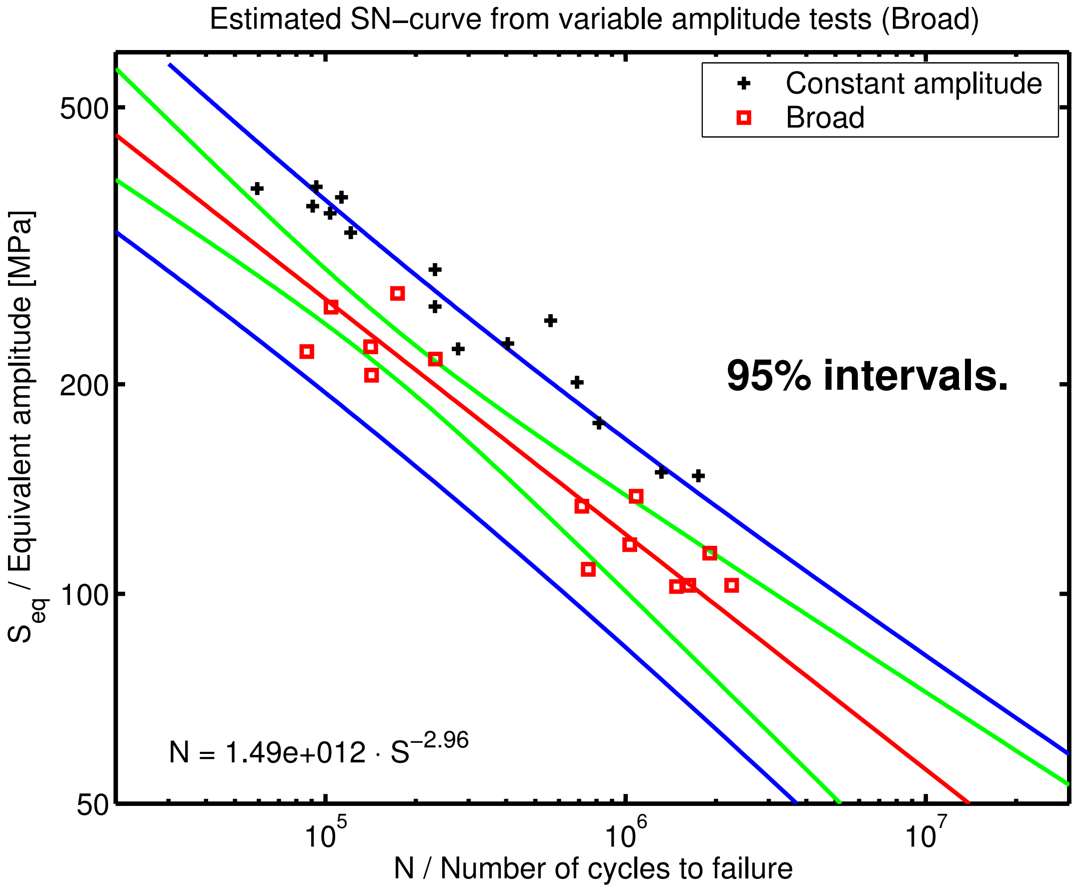


- **Estimated median life.**
- **Confidence interval for median life.**
- **Prediction interval (for future tests).**

- **Relative life, $N_{rel}=N/N_{pred}$, a way to examine systematic prediction errors.**
- **Non-conservative predictions.**
  Broad: N_{rel} = 0.38; (0.31, 0.47)
  Narrow: N_{rel} = 0.53; (0.41, 0.69)
  PM mod: N_{rel} = 0.46; (0.37, 0.59)

Example: Agerskov

Estimated SN-curve from Broad, prediction for CA.

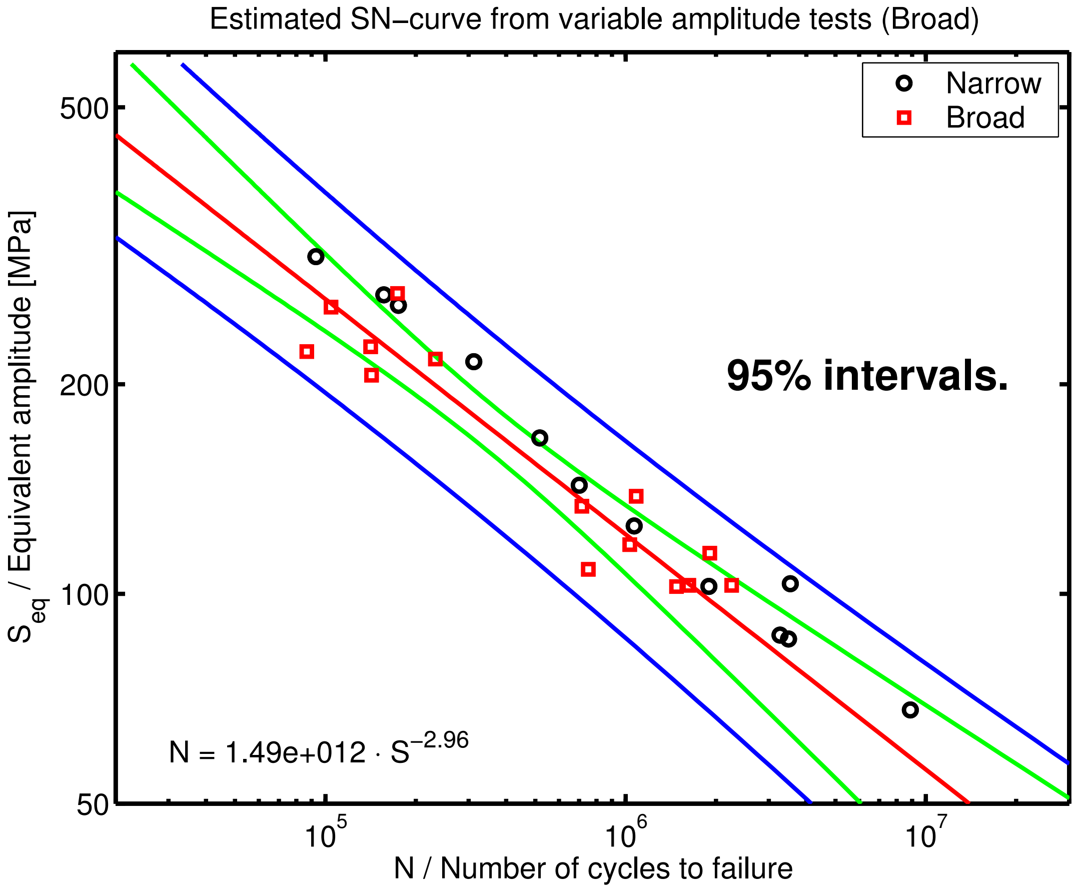


- **Estimated median life.**
- **Prediction interval.**

- **Conservative predictions.** N_{rel} **= 2.57; (1.82, 3.63)**
- **No statistically significant difference for the damage exponent β.**

Example: Agerskov

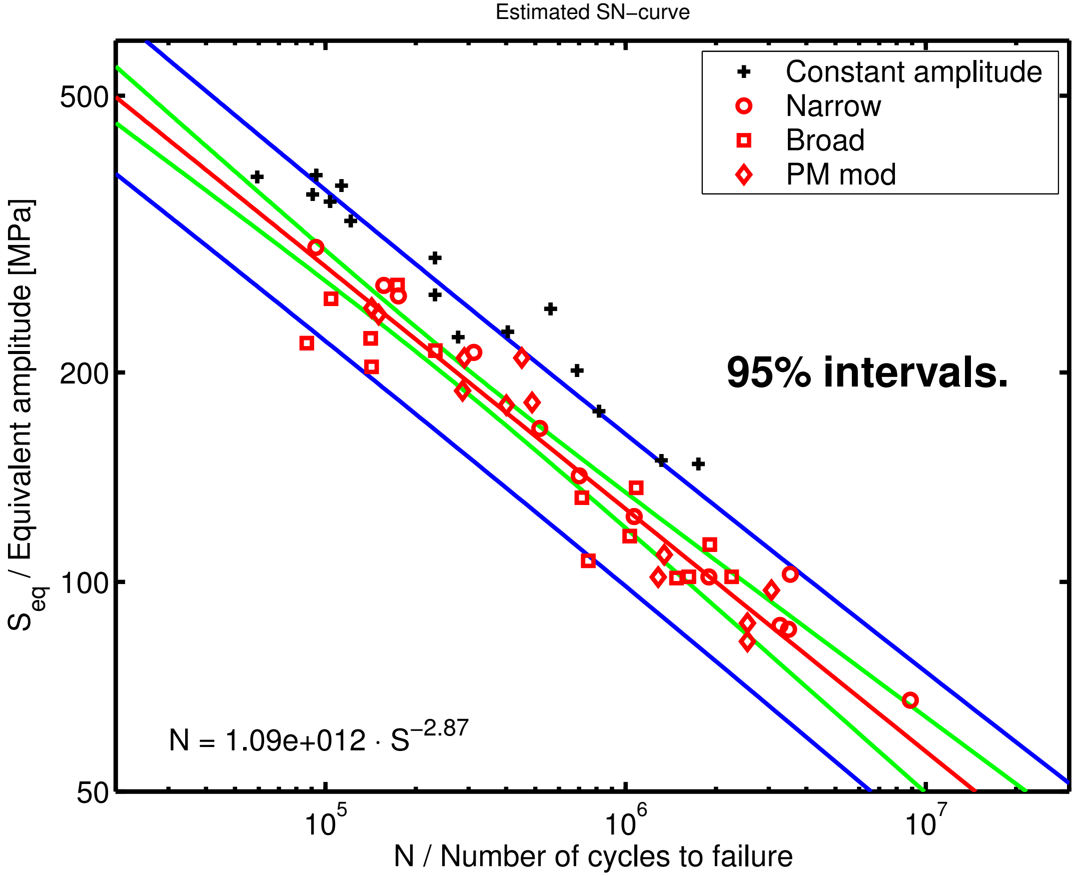
Estimated SN-curve from Broad, prediction for Narrow.



- **Estimated median life.**
- **Prediction interval.**
- **No statistically significant systematic errors.**
  N_{rel} = 1.42; (0.98, 2.05)
- **Prediction based on CA gives systematic errors.**
  N_{rel} = 0.53; (0.41, 0.69)
- **Prediction for PM mod:**
  N_{rel} = 1.23; (0.86, 1.74)

Example: Agerskov

Estimated SN-curve from all VA, prediction for CA.



- **Estimated median life.**
- **Prediction interval.**
- **Possible to combine different types of load spectra when estimating the SN-curve.**
- **Systematic prediction errors.** **N_{rel} = 2.11; (1.69, 2.64)**
- **No difference seen in β!**

Extended Fatigue Model – Mean Value Influence

- **Mean value correction**
  - ***Include mean value correction when calculating S_{eq}.***
  - ***Use existing models, e.g. mean-stress-sensibility (Schütz, 1967)***

$$S_{eq} = \sqrt[\beta]{\sum \nu_j \left(s_{a,j} + M \cdot s_{m,j}\right)^{\beta}}$$

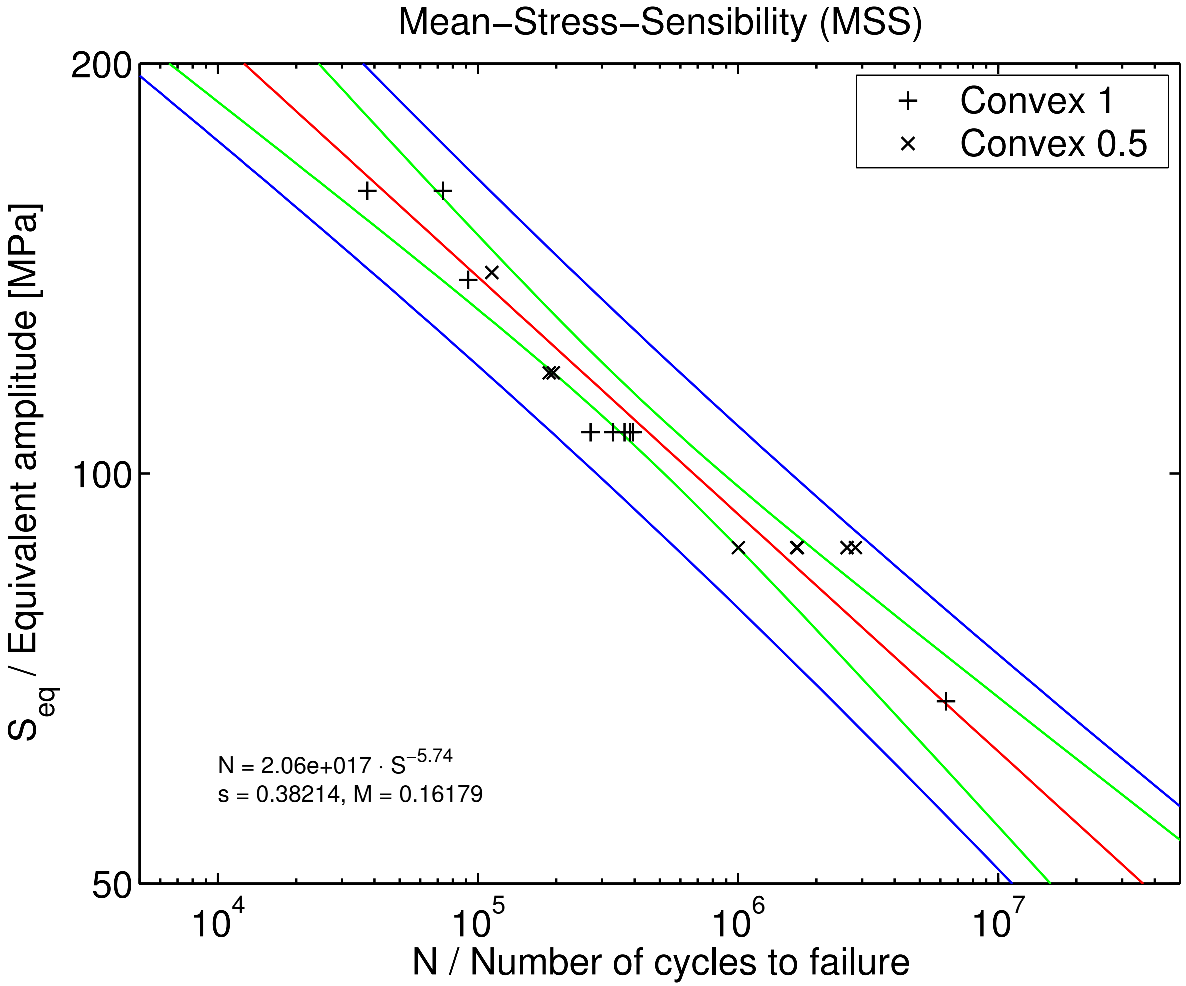
  - ***Possible to estimate the correction M together with SN-curve.***

- **Crack closure models**
  - ***Include crack closure when calculating S_{eq}.***

$$S_{eq} = \sqrt[\beta]{\sum \nu_j \left(S_{\max,j} - S_{op,j}\right)^{\beta}}$$

  - ***Use existing models***
    - **variable closure level, or**
    - **constant closure level.**
  - ***Possible to estimate a constant level S_{op} together with SN-curve.***

SP: Convex spectrum – Mean stress correction



Mean-Stress-Sensibility

$S_a' = S_a + M \cdot S_m$

Uncertainty in parameters

- **$\beta = 5.74$;**
  $4.74 < \beta < 6.75$ (95%)
- **M = 0.17;**
  0.073 < M < 0.251 (95%)

- **Scatter is reduced from s=0.51 to s=0.38.**

- **We should include the extra parameter M.**
- **Optimal model complexity?**

Conclusions

- **Estimation of SN-curve from spectrum tests.**
  - *Methodology based on equivalent load, S_{eq}.*
  - *Can combine different types of load spectra.*
  - *Analysis of uncertainties in estimates and predictions.*
  - *Possible to distinguish systematic deviances form random variations.*
    - Difference between CA and VA? Yes, often! (same β)
    - Difference between load spectra? No, for Agerskov!
    - Influence from mean value, irregularity, or level crossings?
  - *Extensions – Mean value influence.*

- **Further work.**
  - *Combined estimation of SN-curves for CA and for VA (same slope β).*
    - Estimate two SN-curves (CA and VA) at the same time, with some common parameters.
    - Parameters: $(\alpha_{CA}, \alpha_{VA}, \beta, \sigma)$ "different α"
    - Parameters: $(\alpha_{CA}, \alpha_{VA}, \beta, \sigma_{CA}, \sigma_{VA})$ "different α and σ"
  - *Different classes of service loads.* *same slope β ,different α ?*