Workshop Advances in Numerics for S(P)DEs
Chalmers University of Technology & University of Gothenburg

David Cohen & Annika Lang

October 14–18, 2024

Program, titles, and abstracts

Program

Monday	14.10.24	Tuesday	15.10.24	Wednesday	16.10.24
10:20–10:30	Opening				
10:30–11:00	Annika Lang	09:00–09:30	Xinyu Chen	09:00–09:30	Ziyi Lei
11:30–12:30	Lunch (wraps)	09:30–10:00	Tonghe Dang	09:30–10:00	Björn Müller
		10:00–10:30	Coffee break	10:00–10:30	Coffee break
		10:30–11:00	Akash Sharma	10:30–11:00	Yibo Wang
		11:00–11:30	Jianbo Cui	11:00–11:30	Guanlin Yang
		12:00–13:00	Lunch (Kårhuset)	12:00–13:00	Lunch (Kårhuset)
				18:30–	Conference dinner

Thursday	17.10.24	Friday	18.10.24
09:30–10:00	Andrea Papini	09:30–10:00	Johan Ulander
10:00–10:30	Photo and coffee break	10:00–10:30	Coffee break
10:30–11:00	Øyvind Stormark Auestad	10:30–11:00	Ge Liang
11:00–11:30	Xiaojie Wang	11:00–11:30	Fengshan Zhang
11:30–12:00	Wanrong Cao	11:30–	Closing and wraps
12:30–13:30	Lunch (Ooto)		

The afternoons are reserved for scientific discussions.

All talks take place at: room MVL15, Mathematical Sciences, Campus Johanneberg.

The conference dinner takes place at Familjen, Arkivgatan 7.

Titles and abstracts of talks

Øyvind Stormark Auestad (NTNU)

Nested finite element approximation of parabolic SPDEs with Whittle–Matérn noise

Abstract: We propose a new type of fully discrete finite element approximation of a class of semilinear stochastic parabolic equations with additive noise. Our discretization differs from the ones typically considered, in that we employ a nested finite element approximation of the noise. This is well suited for dealing with covariance operators defined in terms of (negative powers of) elliptic operators, like that of Whittle–Matérn random fields. We derive strong and pathwise convergence rates for our proposed discretization, and our results are supported by numerical experiments.

Wanrong Cao (Southeast University)

Finite Difference Method for Stochastic Cahn–Hilliard Equation Driven by A Fractional Brownian Sheet

Abstract: The stochastic Cahn–Hilliard equation driven by a fractional Brownian sheet provides a more accurate model for correlated space-time random perturbations. This study delves into two key aspects: first, it rigorously examines the regularity of the mild solution to the stochastic Cahn–Hilliard equation, shedding light on the intricate behavior of solutions under such complex perturbations. Second, it introduces a fully discrete numerical scheme designed to solve the equation effectively. This scheme integrates the finite difference method for spatial discretization with the tamed exponential Euler method for temporal discretization. The analysis demonstrates that the proposed scheme achieves a strong convergence rate of $O\big(h^{1-\epsilon}+\tau^{H_1-\frac{1}{8}-\frac{\epsilon}{2}}\big)$, where ϵ is an arbitrarily small positive constant, providing a solid foundation for the numerical treatment of such equations.

Xinyu Chen (Chinese Academy of Sciences)

Superiority of stochastic symplectic methods via the law of iterated logarithm

Abstract: The superiority of stochastic symplectic methods over non-symplectic counterparts has been verified by plenty of numerical experiments, especially in capturing the asymptotic behaviour of the underlying solution process. This talk aims to theoretically investigate the superiority from the perspective of the law of iterated logarithm, taking the linear stochastic Hamiltonian system in Hilbert space as a test model. Based on the time-change theorem for martingales and the Borell–TIS inequality, we first prove that the upper limit of the exact solution with a specific scaling function almost surely equals some non-zero constant, thus confirming the validity of the law of iterated logarithm. Then, we prove that stochastic symplectic fully discrete methods asymptotically preserve the law of iterated logarithm, but non-symplectic ones do not. This reveals the good ability of stochastic symplectic methods in characterizing the almost sure asymptotic growth of the utmost fluctuation of the underlying solution process. Applications of our results to the linear stochastic oscillator and the linear stochastic Schrödinger equation are also presented.

This is a joint work with Chuchu Chen, Tonghe Dang, and Jialin Hong.

Jianbo Cui (The Hong Kong Polytechnic University)

Wasserstein Hamiltonian Flow and Its Structure Preserving Numerical Scheme

Abstract: We study discretizations of Hamiltonian systems on the probability density manifold equipped with the L_2-Wasserstein metric. For low dimensional problems, based on discrete optimal transport theory, several Wasserstein Hamiltonian flows (WHFs) on graph are derived. They can be viewed as spatial discretizations to the original systems. By regularizing the system using Fisher information, we propose a novel regularized symplectic scheme which could preserve several desirable longtime behaviors. Furthermore, we use the coupling idea and WHF to propose a supervised learning scheme for some high-dimensional problem. If time permits we will talk about more details on solving high-dimensional Hamilton–Jacobi equation via the density coupling and supervised learning.

Tonghe Dang (Chinese Academy of Sciences)

Probabilistic limit behaviors of numerical discretizations for time-homogeneous Markov processes

For the general time-homogeneous Markov process, do numerical discretizations exactly preserve its probabilistic limit behaviors, in particular the strong LLN and the CLT? In this work, we give a positive answer to the question by proposing a unified approach to investigating probabilistic limit behaviors of numerical discretizations of time-homogeneous Markov processes. Once the properties of strong mixing and convergence for numerical discretizations are satisfied, it is shown that the time-averages of numerical discretizations converge to the ergodic limit in the almost surely sense and that

the normalized time-averages converge in distribution to a normal distribution. The limits coincide with the ones for the underlying Markov process. Our results can be applied to numerical discretizations for a large class of stochastic differential equations, including stochastic Allen–Cahn equation and stochastic functional differential equations.

This is a joint work with Chuchu Chen, Jialin Hong, and Guoting Song.

Annika Lang (Chalmers and GU)

Stochastic partial differential equations on surfaces and evolving random surfaces: a computational approach

Abstract: Looking around us, many surfaces including the Earth are no plain Euclidean domains but special cases of Riemannian manifolds. One way of describing uncertain physical phenomena on these surfaces is via stochastic partial differential equations. In this talk, I will introduce how to compute approximations of solutions to such equations and give convergence results to characterize the quality of the approximations. Furthermore, I will show how these solutions on surfaces are a first step towards the computation of time-evolving stochastic manifolds.

Ziyi Lei (Chinese Academy of Sciences)

Numerical approximation of the invariant measure for a class of stochastic damped wave equations

Abstract: We study a class of stochastic semilinear damped wave equations driven by additive Wiener noise. Owing to the damping term, under appropriate conditions on the nonlinearity, the solution admits a unique invariant measure. We apply semi-discrete and fully-discrete methods in order to approximate this invariant measure, using a spectral Galerkin method and an exponential Euler integrator for spatial and temporal discretization respectively. We prove that the considered numerical schemes also admit unique invariant measures, and we prove error estimates between the approximate and exact invariant measures, with identification of the orders of convergence. To the best of our knowledge, this is the first result in the literature concerning the numerical approximation of invariant measures for stochastic damped wave equations.

This is a joint work with Charles-Edouard Bréhier and Siqing Gan.

Ge Liang (Chinese Academy of Sciences)

Long-time weak convergence analysis of a semi-discrete scheme for stochastic Maxwell equations

Abstract: In this talk, we focus on investigating the weak convergence of the semi-implicit Euler scheme for stochastic Maxwell equations on the infinite time horizon. Based on the properties of the Maxwell operator, we first analyze the regularities of transformed Kolmogorov equation associated with the stochastic Maxwell equations. Then by constructing an adapted continuous auxiliary process of the semi-implicit Euler scheme, we prove that the long-time weak convergence order of the scheme is one, which is twice the strong convergence order. Lastly, we give some applications of the weak convergence result.

This is a joint work with Chuchu Chen and Jialin Hong.

Björn Müller (Chalmers and GU)

Hölder regularity and spectral simulation of Q-fractional Brownian motion on the sphere

Abstract: We consider, in analogy to the Q-Wiener process commonly used in SPDE theory, a Q-fractional Brownian motion (Q-fBm) on the (d-dimensional) sphere. This Q-fBm defines a spatio-temporal stochastic process on the sphere with spatial covariance operator Q that follows the dynamics of fractional Brownian motion in time. We show its temporal and space-time Hölder regularity depending on the Hurst parameter H and the angular power spectrum of Q. A spectral approximation based on the Karhunen–Loève expansion reduces the simulation of Q-fBm to simulating independent, scalar-valued fractional Brownian motions. We apply this procedure with different methods that (exactly) simulate or approximate scalar fBm and examine approximation errors in space and time.

Andrea Papini (Chalmers and GU)

Average dissipation for stochastic transport equations with Lévy noise

Abstract: We show that, in one spatial and arbitrary jump dimension, the averaged solution of a Marcus-type SPDE with pure jump Lévy transport noise satisfies a dissipative deterministic equation involving a fractional Laplace-type operator. To this end, we identify the correct associated Lévy measure for the driving noise. We consider this a first step in the direction of a non-local version of enhanced dissipation, a phenomenon recently proven to occur for Brownian transport noise and the associated local parabolic PDE. Moreover, we present numerical simulations, supporting the fact that dissipation occurs for the averaged solution, with a behavior akin to the diffusion due to a fractional Laplacian, but not in a pathwise sense.

Based on a joint work with Franco Flandoli and Marco Rehmeier.

Akash Sharma (Chalmers and GU)

Sampling in bounded domain using stochastic differential equations

Abstract: We will present numerical integrators for reflected and confined SDEs. Of particular interest are the objects: reflected (overdamped) Langevin dynamics and confined (underdamped) Langevin dynamics. We will also discuss about the rate of convergence of presented integrators. Together with examples, we will show how these SDEs can be used for sampling from a desired measure and for non-convex optimization with boundary constraints.

Johan Ulander (Chalmers and GU)

Artificial barriers for stochastic differential equations and for construction of boundary-preserving schemes

Abstract: We develop the novel method of artificial barriers for scalar stochastic differential equations (SDEs) and use it to construct boundary-preserving numerical schemes for strong approximations of scalar SDEs, possibly with non-globally Lipschitz drift and diffusion coefficients, whose state-space is either bounded or half-bounded. The idea of artificial barriers is to augment the SDE with artificial barriers outside the state-space to not change the solution process, and then apply a boundary-preserving numerical scheme to the resulting reflected SDE (RSDE). This enables us to construct boundary-preserving numerical schemes with the same strong convergence rate as the strong convergence rate of the numerical scheme for the corresponding RSDE. Based on the method of artificial barriers, we define a boundary-preserving scheme that we call the Artificial Barrier Euler–Maruyama (ABEM) scheme. We provide numerical experiments for the ABEM scheme and the numerical results agree with the obtained theoretical results.

Xiaojie Wang (Central South University)

Higher order boundary preserving discretization scheme for scalar SDEs

Abstract: In this talk, we consider higher order strong approximation of scalar stochastic differential equations (SDEs) taking values in a domain and having non-Lipschitz coefficients. By combining a Lamperti-type transformation with a semi-implicit discretization approach and a taming strategy, we construct a domain-preserving scheme that strongly converges under weak assumptions. Moreover,

we show that this scheme has strong convergence order 1.5 under additional assumptions on the coefficients of the SDE. In our scheme, the domain preservation is a consequence of the semi-implicit discretization approach, while the taming strategy allows controlling terms of the scheme that admit singularities but are required to obtain the desired order. Our general convergence results are applied to various SDEs from applications, with sub-linearly or super-linearly growing and non-globally Lipschitz coefficients. Numerical experiments are presented to illustrate our theoretical findings.

This talk is based on a joint work with Ruishu Liu and Andreas Neuenkirch.

Yibo Wang (Southeast University)

Approximation of the invariant measure for stochastic Allen–Cahn equation via an explicit fully discrete scheme

Abstract: In this paper we propose an explicit fully discrete scheme to numerically solve the stochastic Allen–Cahn equation. The spatial discretization is done by a spectral Galerkin method, followed by the temporal discretization by a tamed accelerated exponential Euler scheme. Based on the time-independent boundedness of moments of numerical solutions, we present the weak error analysis in an infinite time interval by using Malliavin calculus. This provides a way to numerically approximate the invariant measure for the stochastic Allen–Cahn equation.

Guanlin Yang (Chinese Academy of Sciences)

Inverse random potential scattering for stochastic polyharmonic wave equations

Abstract: In this talk, we mainly discuss the uniqueness of the inverse scattering problem for the random potential involved in stochastic polyharmonic wave equations. The random potential is assumed to be an isotropic generalized Gaussian random field. With limited measurements, we show the unique determination of the correlation strength of the random potential through a single realization of the scattered wave field averaged over the frequency band.

This is a joint work with Jianliang Li, Peijun Li, and Xu Wang.

Fengshan Zhang (Chinese Academy of Sciences)

A new class of splitting methods that preserve ergodicity and exponential integrability for stochastic Langevin equation

Abstract: In this talk, we propose a new class of splitting methods to solve the stochastic Langevin equation, which can simultaneously preserve the ergodicity and exponential integrability of the original equation. The central idea is to extract a stochastic subsystem that possesses the strict dissipation from the original equation, which is inspired by the inheritance of the Lyapunov structure for obtaining the ergodicity. We prove that the exponential moment of the numerical solution is bounded, thus validating the exponential integrability of the proposed methods. Further, we show that under moderate verifiable conditions, the methods have the first-order convergence in both strong and weak senses, and we present several concrete splitting schemes based on the methods. The splitting strategy can be readily extended to construct conformal symplectic methods and high-order methods that preserve both the ergodicity and the exponential integrability, as demonstrated in numerical experiments. Our numerical experiments also show that the proposed methods have good performance in the long-time simulation.

This is a joint work with Chuchu Chen, Tonghe Dang, and Jialin Hong.

List of participants

- Chuchu Chen, Chinese Academy of Sciences
- Xinyu Chen, Chinese Academy of Sciences
- Tonghe Dang, Chinese Academy of Sciences
- Jialin Hong, Chinese Academy of Sciences
- Ge Liang, Chinese Academy of Sciences
- Guoting Song, Chinese Academy of Sciences
- Xu Wang, Chinese Academy of Sciences
- Guanlin Yang, Chinese Academy of Sciences
- Fengshan Zhang, Chinese Academy of Sciences
- Liying Sun, Capital Normal University
- Xiaojie Wang, Central South University
- Wanrong Cao, Southeast University
- Yibo Wang, Southeast University
- Jianbo Cui, The Hong Kong Polytechnic University
- Øyvind Stormark Auestad, NTNU
- David Cohen, Chalmers and GU
- Annika Lang, Chalmers and GU
- Stig Larsson, Chalmers and GU
- Björn Müller, Chalmers and GU
- Andrea Papini, Chalmers and GU
- Akash Sharma, Chalmers and GU
- Johan Ulander, Chalmers and GU

Venue

The workshop takes place at: room MVL15, Mathematical Sciences, Campus Johanneberg

A map of the campus can be found here