Mystery of point charges

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Abstract

We discuss the problem of finding an upper bound for the number of equilibrium points of a potential of several fixed point charges in $\mathbf{R}^n$. This question goes back to J. C. Maxwell and M. Morse. Using fewnomial theory we show that for a given number of charges there exists an upper bound independent of the dimension, and show it to be at most 12 for three charges. We conjecture an exact upper bound for a given configuration of nonnegative charges in terms of its Voronoi diagram, and prove it asymptotically.

Main references

J. C. Maxwell, *A Treatise on Electricity and Magnetism*, **vol. 1**, Republication of the 3rd revised edition, Dover Publ. Inc., 1954.
M. Morse and S. Cairns, *Critical Point Theory in Global Analysis and Differential Topology*, Acad. Press, 1969.
A. Gabrielov, D. Novikov, and B. Shapiro, *Mistery of point charges*, math-ph/0409009.