Läskurs i Analytisk talteori

The aim of this course is to illustrate how the methods of classical real and complex analysis were used to tackle number-theoretic problems during the historical period 1800-1930 (approx.), which may be considered the 'golden age' of analytic number theory. Together with a course in algebraic number theory which includes class field theory, it thus serves the purpose of bringing the student interested in studying number theory at the graduate level to the point where (s)he is in a position to begin approaching the more modern developments.

Students taking the course are expected to have a background comparable with the content of my recent course MAN 640 (algebraisk talteori). In that course I introduced L-functions and proved Dirichlet's theorem on primes in arithmetic progressions, which material may be found in chapters 1-6 of Davenport's 'Multiplicative number theory'.

The course is primarily a reading course, though there will be one two-hour lecture every week. These lectures will, by necessity, be rather sketchy, with the object of giving an overview of a certain topic, such that the interested student may then pursue a more thorough investigation on his/her own, and in consultation with myself. The course will continue over the whole spring semester, and is worth 5 points. Examination will be oral.

The following is an outline of the main topics to be covered. As the course is meant to be an interaction between students and teacher, modifications are possible based on the prior knowledge and interests of the participants.

1. Analytic theory of Dirichlet L-functions : analytic continuation, functional equation, canonical product, prime number theorem, Riemann hypothesis.

Reference : H. Davenport, 'Multiplicative number theory', Springer.

2. Additive problems : Twin primes, Goldbach conjecture, Waring's problem, partitions of n. Hardy-Littlewood circle method and sieve methods.

References : Will probably be a mish-mash with handouts and visits to the library encouraged ! If you can get your hands on a copy of Edmund Landau's 'Vorlesungen über Zahlentheorie', then please do so !

3. Modular forms : Eisenstein series, cusp forms, Hecke operators, L-series, theta series and quadratic forms.

References : T. Apostol, 'Modular forms and Dirichlet series in number theory', Springer, 1976.
J.P. Serre, 'A course in arithmetic', Springer.

N.B.: Students following this course are encouraged to also attend Per Salberger's lectures on elliptic curves.