MATEMATIK Dag : 031020 Tid : 14.15 - 18.15.
Chalmers Tekniska Högskola Hjälpmedel : Inga
Peter Hegarty Vakt : Nalle Puh 772 1000.

Tentamenskriving i Diskret Matematik (TMA 055)

75 poäng på denna tenta och 150 poäng totalt, inkl. poängen från inlämningsuppgifterna, ger godkännt. Dessa gränser kan minskas efteråt.

1 (20p) Suppose you choose 1002 numbers from the set $\{1, 2, ..., 2003\}$. Explain why, among the numbers you chose, there must be two which are relatively prime.

2 (13p+13p) For each integer $n \geq 0$, let D_n denote the number of Dyck paths from $(0, 0)$ to $(2n, 0)$ which never go above the line $y = 2$.

(i) Explain why

$$D_n = \sum_{m=0}^{n-1} D_m, \qquad \forall\, n \geq 0.$$

(ii) Hence, or otherwise (there are other ways to do it), find an explicit formula for D_n (Hint : If you get stuck, compute the first few D_n. The pattern should be quite clear, so you can at least guess the right answer).

3 (26p) For $n \geq 0$, let q_n denote the number of words of length n in the alphabet $\{a, b, c, d\}$ which contain no two consecutive b's. Write down and solve a recurrence relation for q_n.

4 (26p) Let n be an odd integer. Prove that $n^5 - n$ is divisible by 240.

5 (26p) Find all integers x which satisfy

$$\begin{aligned} x &\equiv 2 \pmod 7, \\ x &\equiv 3 \pmod{11}, \\ x &\equiv 4 \pmod{13}. \end{aligned}$$

6 (8p+18p) (i) Compute $\phi(7000)$.
(ii) Let n be an integer which is a product of two distinct odd primes p and q. Assuming Fermat's Theorem for both p and q, state and prove Euler's

Theorem for n.

Obs! Tentan beräknas vara färdigrättad den 27 oktober. Då kan den hämtas i mottagningsrummet mellan kl. 12:30-13:00. Tentamensresultat lämnas också ut per telefon 772 35 09 *efter* kl. 14:00.