MATEMATIK Dag : 031020 Tid : 14.15 - 18.15.
Chalmers Tekniska Högskola Hjälpmedel : Inga
Peter Hegarty Vakt : Nalle Puh 772 1000.

Tentamenskriving i Diskret Matematik (TMA 055)

75 poäng på denna tenta och 150 poäng totalt, inkl. poängen från inlämningsuppgifterna, ger godkännt. Dessa gränser kan minskas efteråt.

1 (20p) Let n, k be non-negative integers. Let $\lambda(n,k)$ denote the number of ways to choose k numbers from the set $\{1, ..., n\}$ such that no two are consecutive. Prove that, if $n \geq 2$ and $k \geq 1$, then

$$\lambda(n,k) = \lambda(n-1,k) + \lambda(n-2,k-1).$$

2 (26p) Solve completely the recurrence relation

$$\begin{aligned} u_0 = u_1 &= 1, \\ u_n = 2u_{n-1} + 3u_{n-2} + 2^n, &\quad \forall\, n \geq 2. \end{aligned}$$

3 (26p) Find the general solution to the Diophantine equation

$$19x + 43y = 3000$$

and hence, or otherwise, find all solutions such that both $x > 0$ and $y > 0$.

4 (26p) Find all integers x which satisfy

$$\begin{aligned} x &\equiv 1 \pmod{11}, \\ x &\equiv 3 \pmod{19}, \\ x &\equiv 5 \pmod{29}. \end{aligned}$$

Also, determine the smallest positive integer solution.

5 (6p+8p+12p) You are referred to the graph G in Figure 1[1].

(i) Describe a Hamilton cycle in G, or explain why none exists, if such is the case.

[1] which will be drawn on the board at Monday's lecture.

(ii) Determine a minimal spanning tree in G (write down the order in which you choose the edges).

Now direct the graph as follows : all vertical edges are to be directed upwards, and all other edges from left to right. Call this directed graph G^*.

(iii) In G^*, apply Dijkstra's algorithm to find a shortest path from A to K. OBS! Write down all your calculations.

6 (26p) Prove that there are infinitely many right-angled triangles whose side-lengths are all integers.

Obs! Tentan beräknas vara färdigrättad den 27 oktober. Då kan den hämtas i mottagningsrummet mellan kl. 12:30-13:00. Tentamensresultat lämnas också ut per telefon 772 35 09 *efter* kl. 14:00.