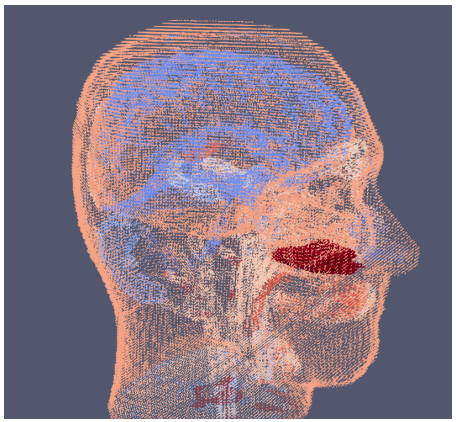
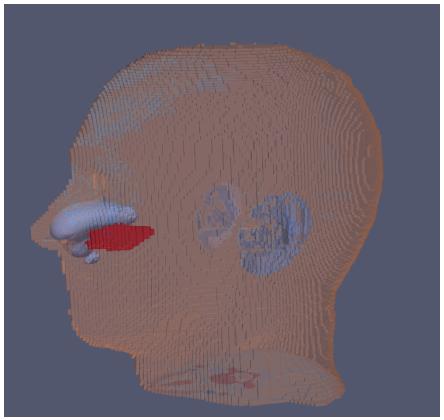
Project "Adaptive algorithms for inverse problems with applications in medical imaging"

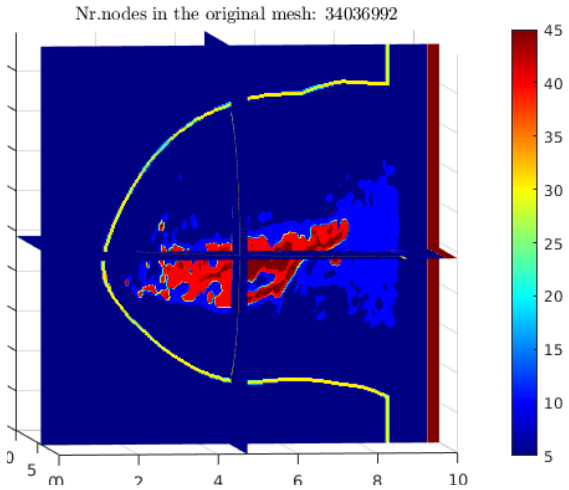
Reference number PAR 2024/1137

Coefficient Inverse Problems: medical applications

Figure: left, middle: microwave hyperthermia in cancer treatment; right: breast cancer detection using microwaves. https://doi.org/10.1515/jiip-2020-0102







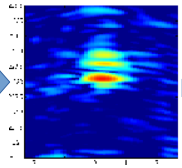
- Breast cancer, land mines, oil prospecting, ability to see through the walls and construction of "invisible materials" can all be modelled and computed using different types of wave equations: acoustic, elastic or electromagnetic.
- In the mathematical literature, these questions are called the coefficient inverse problems. A coefficient inverse problem for a given PDE aims at estimating a spatially distributed coefficient of the model PDE using measurements taken on the boundary of the domain of interest.

Coefficient Inverse Problems: main steps in solution

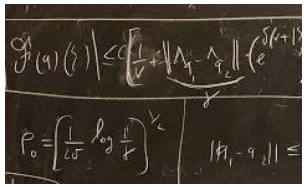
Coefficient Inverse Problems for PDE

A coefficient inverse problem for a given partial differential equation (PDE) aims at estimating a spatially distributed coefficient of the model PDE using measurements taken on the boundary of the domain of interest.

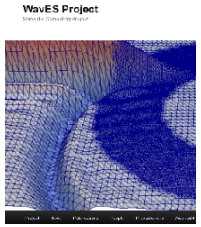
Use collected data. Data is generated by different type of waves (acoustic, elastic, electromagnetic)



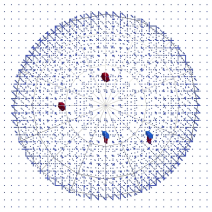
Extract image data for analysis.



Develop new mathematical methods for reconstruction of spatially distributed coefficients in the model PDE which describes the physical process.



Implement new methods using fast and robust numerical methods.



Visualize and analyze obtained reconstructions. Estimate error in the reconstruction.

The main objective of the project

The main ideas of the project will be based on the research presented in [1,2,3].

- The main purpose of the project is to develop new adaptive non-local numerical method for solving the multidimensional coefficient inverse problems (MCIP's) in the framework of hyperbolic equations.
- Development of Carleman estimates to prove global uniqueness of reconstructed parameters in MCIP's is one of the main challenges of the project.
- New non-local methods for hyperbolic MCIP's (for wave and Maxwell's equations) will allow us to construct non-local algorithms to detect skin and breast tumours in real time.

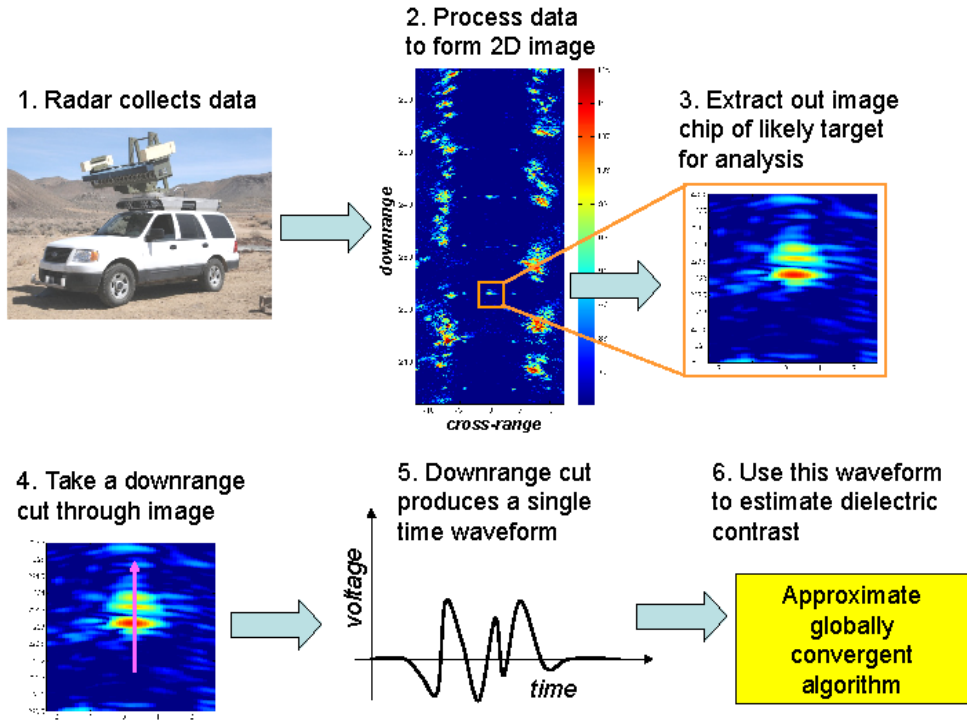
[1] M. Asadzadeh and L. Beilina, A posteriori error analysis in a globally convergent numerical method for a hyperbolic coefficient inverse problem, *Inverse Problems*, 26, 115007, 2010.
[2] L. Beilina and M. V. Klibanov, *Approximate global convergence and adaptivity for Coefficient Inverse Problems*, Springer, New York, 2012
[3] L. Beilina, E. Lindström, An Adaptive Finite Element/Finite Difference Domain Decomposition Method for Applications in Microwave Imaging, *Electronics* 2022, 11(9), 1359; https://doi.org/10.3390/electronics11091359

Example: two-stage numerical procedure of [BK]

MCIP: determination of refractive indices of materials in non-conductive medium. We have used two-stage procedure:

Stage 1. Approximately globally convergent numerical method provides a good approximation for the exact solution.

Stage 2. Adaptive Finite Element Method refines it.



[BK] L. Beilina and M. V. Klibanov, *Approximate global convergence and adaptivity for Coefficient Inverse Problems*, Springer, New York, 2012