Assignment 1

TMA 682, Applied Mathematics

1. Solve the initial value problem below, for $t > 0$,

$$y''(t) - 2y'(t) + y(t) = 1, \quad y(0) = -1, \quad y'(0) = 1.$$

2. Find the Fourier series of the 2π-periodic function

$$f(x) = x^2, \quad 0 < x < 2\pi.$$

   Use it to calculate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots.$$

3. Find the solution $u(x,t)$ of the inhomogeneous wave equation

$$\begin{gathered} u_{xx} = u_{tt} - 1, \quad 0 < x < 1, \quad t > 0, \\ u(0,t) = 0, \quad u(1,t) = 2, \\ u(x,0) = 0, \quad u_t(x,0) = 0. \end{gathered}$$