Assignment 1

TMA682, Applied Mathematics

1. Solve the initial value problem below, for $t > 0$,

$$y''(t) + 4y(t) = 3, \quad y(0) = 1, \quad y'(0) = 5.$$

2. Find the Fourier series of the 2π-periodic function

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi. \end{cases}$$

   Use it to calculate the sum

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \dots.$$

3. Find the solution $u(x, t)$ of the inhomogeneous Laplace equation

$$\begin{cases} u_{xx} + u_{yy} = y, & x > 0, & 0 < y < 1, \\ u(x, 0) = u(x, 1) = 0, & x > 0, & \\ u(0, y) = y - y^3, & & 0 < y < 1, \\ u \quad \text{bounded} & \text{as } x \to \infty. & \end{cases}$$